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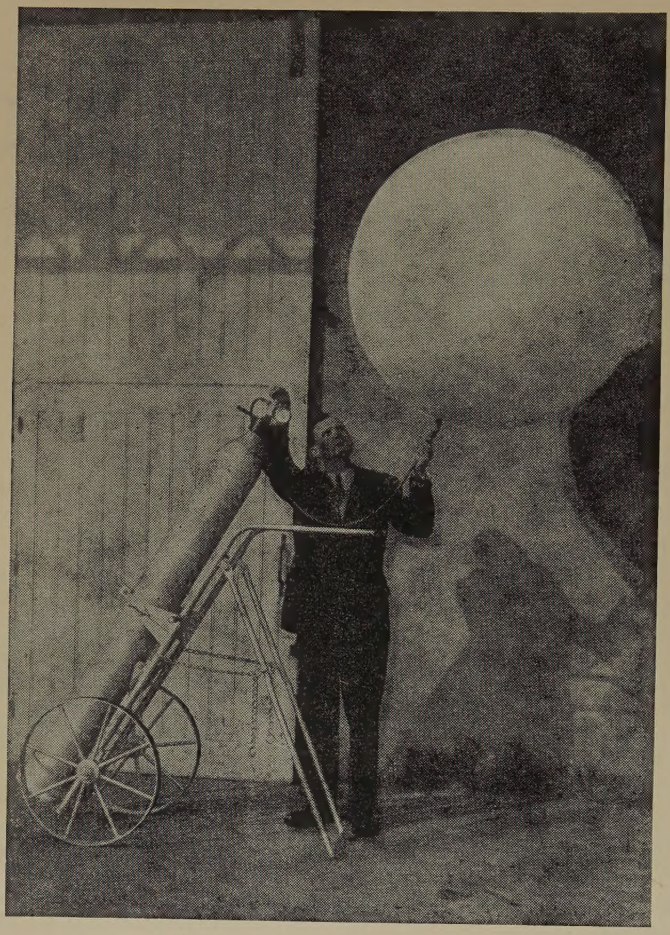

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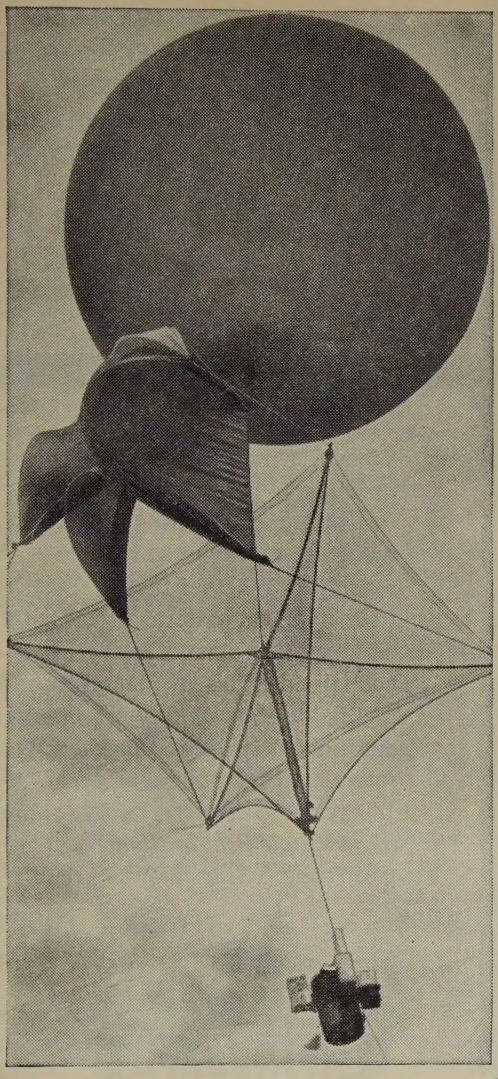

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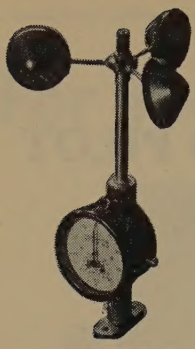

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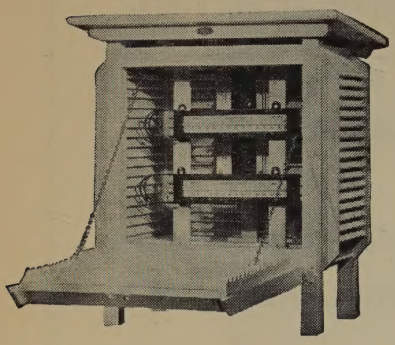

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Synoptic and dynamic aspects of climatic change

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(Manuscript received 8 December 1958)

SUMMARY

The changing pattern of world climate during the last hundred years is compared with the inferred pattern of the Pleistocene. Relatively cool climates at high latitudes appear to have been associated much of the time with reduced tropical sea-surface temperatures, a lower snow-line on tropical mountains, reduced aridity on the desert fringes and increased storm activity along continental east coasts in middle latitudes. Relatively warm climates at high latitudes tend to be associated with the opposite complex of conditions.

Considerations of the water balance suggest that a small temperature decrease would be necessary and sufficient for the re-establishment of glacial conditions in many regions. Increased snowfall would help but is not essential. Synoptic considerations indicate that glaciation was associated necessarily with strong upper westerlies at relatively low latitudes over the continents and probably with vigorous anticyclones over the subtropical oceans.

An increased infra-red cooling rate in the upper air, particularly over the tropics, could produce all the necessary conditions for the establishment of a glacial/pluvial phase. Possible causes for changes in the infra-red cooling rate are discussed.

1. INTRODUCTION

Climate depends on the quality and quantity of solar radiation, on the Earth's orbital and rotational motion, on the constitution of the atmosphere and on the configuration of the Earth's surface. A change in each of these factors must produce a change in the general circulation and in the climate. This has been discussed in some detail by Brooks (1949) and in Shapley (1953).

For the purpose of this paper, climatic fluctuations will be defined as events which affected the whole or the major part of the globe for periods which are long compared to the life of the most persistent individual cyclones or anticyclones. This means that the effect of the fluctuations should be consistently recognizable for a period of at least two years, though that is obviously an arbitrary limit. There is no upper limit. Fluctuations of varying periods and amplitudes probably occurred throughout the Earth's history. They had spectacular phenomenalogical effects during the Pleistocene, but they can also be found in instrumental records from the last hundred years.

In the present study, I have tried to search for a physically plausible explanation, based on a minimum number of assumptions, which would be consistent with all the different types of climates known to have occurred during the last half-million years. A consistent explanation is not necessarily true. It could only be justified at all, if the relatively small changes of recent times can be shown to have had basically the same character as the long-term glacial/interglacial sequence of the Pleistocene. This appears to have been the case.

2. WORLD-WIDE CLIMATIC FLUCTUATIONS DURING THE LAST 100 YEARS

During the last 100 years weather elements have been observed for the first time with scientific instruments on all continents except Antarctica. The density, quality and consistency of these observations was not sufficient to demonstrate changes in the climatic

pattern of the world with much detail, nor has all possible information been drawn from the existing records; the following, however, has been learned :

(i) The equatorial rainfall belt tended to be narrower and the arid zone tended to be wider during the first half of this century, than during either the second half of the last century or the immediately passed decade. Evidence of this deduction has been obtained by Butzer (1957) and Kraus (1955, 1958).

(ii) The Australian and North American east coasts up to a latitude of about 45° exhibited the same pattern of change in the precipitation regime as the tropics. Arakawa (1956) has shown that the same happened to a smaller degree along the east coast of Asia. The recent return to somewhat wetter conditions was associated in eastern Australia and in eastern North America with an increased incidence of tropical cyclones. For the Australian coast this has been demonstrated by Brunt and Hogan (1956) and for the American coast by Rodewald (1958).

(iii) There is some evidence of a similar rhythm of change in the monsoon region of South-East Asia and North Australia, but the amplitude of the change was very much smaller there.

(iv) There is no evidence of a world-wide change in precipitation north of 40° lat. However, temperatures at many stations in those latitudes have tended to increase. For example, for Great Britain this was shown by Manley (1951) and Veryard (1956) and for Canada it was demonstrated by Thomas (1955) and Longley (1953). The graphs of the last author also showed, that after a series of relatively warm years, the Canadian Prairie Provinces have tended to become colder again since 1945.

(v) The North Atlantic sea-water may have been warmer on the average in the present century than in the last one. This is suggested by the fact that all coastal stations in the North Atlantic area north of 40° show the highest mean monthly air temperature increase in autumn and winter and the lowest in spring and early summer. The heating effect of the sea could have caused this pattern. Increased sea-water temperatures permitted longer navigation seasons in Spitzbergen and other arctic ports. Ships' traffic became possible for several months of the year along the north-east passage.

(vi) Glaciers in many parts of the world have tended to retreat. This retreat was most noticeable at the beginning of the century. During the last 15 years or so it appears to have slowed down in many cases.

It will now be necessary to see how far the pattern of these changes was paralleled in earlier periods.

3. Pleistocene pattern

During the last glacial period a continuous sheet of ice covered much of North America, extending east of the Mississippi to about 40°N. lat. The more or less contemporaneous Würm glaciation in Europe reached its greatest continuous southward extension to about 50°N over the Ukraine. Many of the higher mountain ranges south of this latitude were covered at the same time by isolated ice fields. The Würm glaciation was preceded by an interglacial which is estimated by Broeker, Turekian and Heezen (1958) to have lasted about 80,000 years. This period was warmer than today, as indicated by fossil plants, and the Arctic Ocean appears to have been ice-free almost continuously.

About 12,000 years ago the glaciers had retreated from most of Scandinavia. This end of the Ice Age was followed by a period with higher temperatures than today in Europe and in the north-eastern United States of America. Deevey and Flint (1957) call this period the Hypsithermal. In Europe it lasted from about 7,000 B.C. to 1,000 B.C. The Arctic Ocean was probably ice-free during most of the Hypsithermal and, as a result, the sea level may have been somewhat higher than today.

Pleistocene glaciation was not restricted to the higher latitudes. Moraines can be found in high tropical mountains up to 3,000 ft below the present limits of glaciation. The extension of the equatorial glaciers can be inferred to have been contemporaneous with the extratropical glaciation. At present the equatorial snow-line coincides roughly with the mean freezing level at a height of about 15,500 ft. During periods of extended equatorial glaciation the snow-line, and therefore the freezing level, appears to have been

at about 13,000 ft. As the temperature lapse rate of the equatorial air is always close to the saturated adiabatic-lapse rate, Flohn (1952) concluded that the sea-level temperature, and by inference the sea-surface temperature, during glaciation was about 4° to 5° lower than it is today.

Past sea-surface temperatures can be estimated and dated with the help of deep-sea sediments. From cores obtained at 21°N and 17°N in the eastern Atlantic it was deduced on ecological grounds by Schott (1952) that Würm sea-surface temperatures were substantially lower than today. Using radio-carbon dating and $0^{16}/0^{18}$ ratio temperature determinations, Rubin and Suess (1955) showed also that water temperatures in the Caribbean were about 5°C lower than at present, until about 12,000 years ago when they began to increase suddenly. This result has been further confirmed and extended by Ericksson, Broecker, Kulp and Wollin (1956), who found from nine correlated samples of bottom silt, obtained from various parts of the North Atlantic between 0° and 41°N, that the ocean waters must have begun to warm about 13,000 to 12,000 years ago. This warming reached its peak rate about 11,000 years ago, at the end of the Ice Age. The colder surface temperature before that time must have been associated with a lower equatorial snow-line, and therefore an extension of the equatorial mountain glaciers during the last glacial period.

The changes in glaciation and temperature on the tropical mountains and in the temperate zone were associated with changes in the rainfall regime at low latitudes. Flint (1959) defines a pluvial period as ' a climatic regimen of sufficiently long duration to be clearly recorded in physical organic evidence and having an average effective rainfall distinctly greater than that of today in the same area.' A period with an average effective rainfall recognizably less than that of today, or distinctly less than that of a preceding or following pluvial period is called non-pluvial. By this definition, a temporary decrease of aridity in a desert region can be called a pluvial, even if conditions are still very dry compared with other parts of the world.

Periods of greater aridity than today are known to have occurred in the anticyclonic sub-tropical belt. Büdel (1949) found that these fluctuations had the character of meridional expansions and contractions of the arid zone. This was deduced on the basis of geological data, before the similar character of recent fluctuations along the fringes of the arid zone had been documented in detail. During the dry or non-pluvial periods, dunes were formed in regions which today are fixed by vegetation. Investigations of these changes in the Sahara have been summarized by Butzer (1957), who concludes that the margins of the desert were drier than today and exposed to sustained wind action during the Pleistocene interglacial periods and again during the postglacial Hypsithermal.

In Australia, the region of south-western New South Wales and northern Victoria which today is an agricultural area with a mean rainfall of about 14 in., has been shown by Butler (1958) to have passed through several open desert phases. The same region has also been very strongly affected by the recent changes described in the preceding chapter.

Pluvials are characterized mainly by higher lake levels. It has been established that these coincide in the south-western United States of America with the last Ice Age. For the drier parts of south-eastern Australia, Gill (1955) deduces from somewhat problematic radio-carbon datings, that wetter conditions prevailed during a period which was ' at least ' 12,000 years ago, that is, at the time of the Würm glaciation. Another relatively wet period, with lake levels in south-east Australia higher than today, began about 3,500 years ago, corresponding to the cool wet sub-Atlantic period in Europe and to renewed glacial advance in the mountains of the western United States of America reported by Rubin and Suess (1955). These wet periods are separated by a warmer and much more arid phase associated with a higher sea-level near Melbourne about 5,000 years ago. This phase is related by Gill to the European ' climatic optimum ' or Hypsithermal and the disappearance of the Arctic ice.

The existence of pluvial and non-pluvial periods in the equatorial zone has been deduced from the variations in the level of the East African lakes. These variations cannot be dated with confidence. However, it seems intrinsically probable, although

not certain, that high equatorial lake levels coincided with extended glaciation on the mountains and with pluvial periods in the sub-tropics along the desert margins. This would make them contemporaneous with the Ice Ages. Low lake-levels probably coincided with the extension of the arid zone during the interglacials. Some possible physical reasons for this assumption will be discussed in Sections 5 and 6.

To summarize, glacial ages at temperate latitudes were apparently associated with a lower equatorial snow-line, lower sea temperatures even in the tropics, a narrower arid zone and probably a pluvial phase in the tropics. Interglacials were mostly associated with higher tropical sea temperatures, higher sea levels, a wider arid zone and probably non-pluvials. The variations between these phases is similar to those observed during the last 100 years, where years of accelerated glacier recession have also been associated with a narrowing of the equatorial rainbelt and with a shortening of the tropical rainy season.

4. The water balance of a snowpack or ice-sheet

The water balance within a vertical column of unit cross-section through a snowpack or ice-sheet can be expressed as follows :

Change of mass = $\pm$ horizontal convergence or divergence of glacial movement
$\pm$ wind deposition or deflation
$+$ snowfall and sublimation
$+$ retention of free water from rain, condensation and melting
$-$ melt
$-$ evaporation

Snow can retain about ten per cent of its own weight as free water. If the horizontal transport of ice and snow is negligible, the snowpack or ice-sheet must be depleted when its free water content is up to capacity and when the melt or evaporation losses exceed the snowfall and sublimation.

Both melting and evaporation require heat which may be supplied through the snow-surface in four different ways.

1. By molecular and turbulent conduction of sensible heat from the air.
2. By an excess of incoming over outgoing radiation.
3. By condensation or sublimation of water vapour from the air upon the surface.
4. By precipitation which is warmer than the surface.

The fourth process is generally the least important. The relative importance of the three others varies with the location and the weather conditions. Heating by condensation can be relatively large in regions where warm, moist, air masses tend to flow over snow surfaces. Heating by radiation is most effective on clear days in low latitudes particularly when the surface consists of old crystalline snow which has a relatively low albedo.

The amount of snowmelt which can be induced by turbulent downward conduction of heat, after the snowpack has been heated to melting temperature, has been investigated by Light (1941) on the basis of earlier studies by Sverdrup (1934). As a first approximation, the monthly amount of water M in centimetres which is being melted by heat supplied from the air can be estimated following Light by an equation of the form

$$M = 1{\cdot}2\, A\bar{v}\, T \qquad . \quad . \quad . \quad . \quad (1)$$

Where $\bar{v}$ is the mean monthly wind velocity at anemometer height in m sec^{-1}, T the mean monthly temperature in °C and A is a basin factor which depends on the nature of the catchment and also possibly on the wind. In general, A varies between 0·4 and unity. Eq. (1) is based on controversial simplifying assumptions, notably the validity of a logarithmic wind change with height, and the absence of a correlation between wind velocity

and temperature. These assumptions are not always realized. However, the equation has been found to give reasonable approximation in practice. Snow cannot accumulate if M exceeds the total snowfall and sublimation. If M is smaller, then the matter may depend on the three other processes mentioned above.

It is of some interest to apply these considerations to data from various climatic regions. In the centre of the Pleistocene ice sheets conditions then may not have been very different from what they are today in the Antarctic. With summer temperatures below freezing, there can be no appreciable melt. Snow accumulation is balanced by glacial movements, wind erosion and evaporation. The colder the air, the smaller is the amount of precipitable moisture which it can contain. Heavy snowfalls do not occur therefore at very low temperatures. A decrease of Antarctic temperatures would probably result in a reduction of the already scanty precipitation. On the other hand, a moderate increase of the temperature could be associated with heavier snowfalls and increased accumulation. Only if the temperature rose high enough to be above melting point for an appreciable part of the year could reduction of the present ice-cap be expected.

Consider next a place like Churchill on the western shore of Hudson's Bay, which is deep in the interior of the former North American ice-sheet. Although Churchill's mean annual temperature between 1931 and 1940 was only $-8°C$, it has no lasting snow cover. The mean recorded summer wind speed is $5{\cdot}4\ \mathrm{m\ sec^{-1}}$. Over the open country of that region the basin factor in Eq. (1) should be relatively high; in fact, it is likely to approach unity. It may be assumed for argument's sake that $A = 0{\cdot}8$ and that the mean wind strength is no more than $1\ \mathrm{m\ sec^{-1}}$. Although this leads almost certainly to an underestimate of the actual turbulent heat conduction towards the snow, it is found that it is sufficient with present Churchill summer temperatures, to melt all the winter snows, even if no heat was contributed by the three other processes mentioned above. Snow and ice might possibly accumulate if Churchill's present small mean winter precipitation was at least trebled provided summer temperatures remained the same. However, accumulation would also be caused almost certainly by a drop of 5°C in mean monthly temperatures even if precipitation were halved, because this would reduce the melting season to a negligible duration.

In warmer regions, summer temperature becomes an increasingly more critical factor. At Haparanda, not very far from the centre of the former Scandinavia ice-sheet the mean annual temperature for the years 1860–1940 was 0·8°C. There is a significant negative correlation between mean monthly temperature and precipitation for the months of January and February. The most critical conditions, i.e., those most close to the possible establishment of a lasting snow cover, appear to have occurred at Haparanda in 1935, the wettest year on record. A relatively mild winter with heavy falls was followed in that year by an exceptionally cool spring and summer. But turbulent heat conduction from the air would have been sufficient even in those conditions to melt all the winter snows. In actual fact, in addition to conduction, a considerable amount of condensation heat must have been supplied to the snow pack during the wet summer of that year. It can be calculated that no physically conceivable increase of precipitation is likely to produce glaciation in the Haparanda region without a decrease of mean summer temperatures from their present value.

It might be argued that a decrease of summer temperatures could be caused by the formation of extensive glaciers in the neighbouring Scandinavian mountains and that this in turn could be induced by greater precipitation alone. The first part of this argument would be in agreement with considerations developed by Brooks who showed that an ice-cap becomes unstable and begins to grow when its area exceeds about 30,000 sq. mi because of its inherent cooling power upon the adjacent air. The low temperature of an extensive ice-surface is due to radiational cooling and poor heat conduction from below. However, heavy precipitation would be associated with many clouds and therefore with reduced radiational cooling. This might not balance the Brooks effect entirely but it would decrease its efficacy. Also in modern times, secular temperature variations appear to be more significant in high latitudes than rainfall variations. From all this, it is concluded

that summer temperatures had probably a greater effect than snowfall upon the establishment and the disappearance of the Pleistocene ice-sheets.

This is certainly the case if we investigate conditions in still lower latitudes. In the Australian Snowy Mountains, at a latitude of about 36°S, glaciers did occur during the Pleistocene at altitudes above 6,000 ft. The winter snows at that height have been known in recent years to accumulate to a depth of 8 m with an equivalent water content of more than 250 cm. However, spring and summer temperatures are so high and radiation is so strong that the snow cannot last through the summer. An amount of precipitation sufficient to do so with present temperatures would have left geological evidence and is physically hardly possible. On the other hand, a drop of about 3°–4° in the mean summer-air temperatures would allow extensive snowfields to last through some summers and it might permit the re-establishment of glaciers even if the present radiational heat gain remained the same.

To summarize, in the absence of melting, snow tends to accumulate even if very little falls each year. In cold regions, over established ice-fields, precipitation alone may therefore determine the amount of mass which accumulates or is exported by glacial movements. In warmer regions summer temperature becomes an increasingly more critical factor. In particular, the establishment of the Pleistocene ice-sheets probably required summer temperatures which were persistently lower by a few degrees than at present. This conclusion tallies with observed glacier fluctuations in modern times. These have been reported by Flint (1957) to be also a function of the temperature rather than of precipitation.

5. The synoptic aspects of different climatic regimes

The change in the climate of many regions which occurred about the end of last century must have been associated with variations in the mean atmospheric circulation. Unfortunately it is not easy to document these directly. Some deductions can be made, however, from the study of more recent synoptic situations or periods, which happened to be characterized either by a combination of relatively low temperatures over North America and Europe with heavy rainfall in the tropics and along the east coasts or by a combination of the opposite type. Two studies by Ballenzweig (1956) and by Flohn (1952) are particularly relevant.

The apparently greater frequency of tropical cyclones along the North American and Australian east coasts during wet periods was mentioned in Section 2. Ballenzweig showed that during years of much hurricane activity in New England and in the Maritime Provinces of Canada, the mean height of the 700 mb surface over the North Atlantic high-pressure cell tends to be well above average. This is associated with a strong southerly component of the mean 700 mb wind along the coast which must have assisted in transporting moisture towards the north. Ballenzweig's maps, one of which is reproduced as Fig. 1, show also that the same situation would tend to cause the mean north-west component of the wind at the 700 mb level across France into the Mediterranean to be greater than average. Such a situation favours outbreaks of cold air into the Gulf of Lyons and the formation of cyclonic vortices south of the Alps. These depressions travel often subsequently as so-called Vb-cyclones in a meridional direction from the Adriatic towards Finland.

Over the tropical Atlantic, a strong development of the Atlantic high-pressure cell suggests a more vigorous development of the direct meridional Hadley-type circulation with a more active production of kinetic energy. In the upper air this must cause increased westerly acceleration. In the lower troposphere it causes a stronger pressure gradient towards the equatorial trough, stronger surface east winds and therefore increased evaporation. Such a situation favours more frequent or more pronounced 'easterly waves' (Riehl 1954), and an increased transport of moisture towards the equatorial trough, the Caribbean and the American coast, with more frequent or more intense rainfall in those regions.

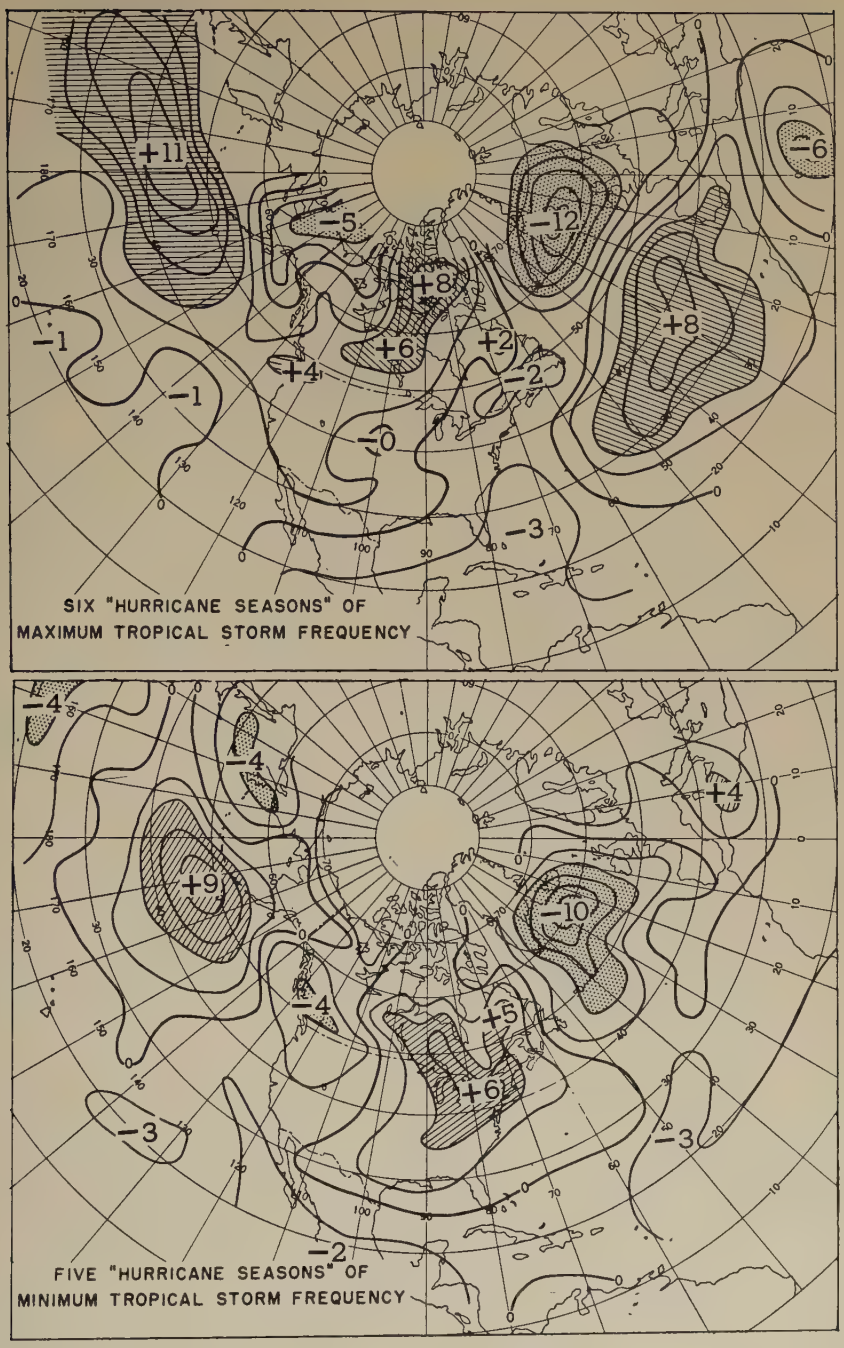

Figure 1. Average anomaly of 700 mb height during hurricane seasons in the New England area (August, September and October) after E. M. Ballenzweig.

Somewhat similar synoptic situations favour heavy rainfall and tropical storms along the southern half of the Australian east coast. The storms move in a poleward direction and are frequently associated with a strong high over the Tasman Sea and the western Pacific; a feature which corresponds to the Azores high-pressure cell in the Atlantic, though it is not as permanent. Radok and Clarke (1958) have noted that cyclogenesis

off the Australian east coast is often preceded by a particularly intense sub-tropical jet stream at low latitudes. On the other hand, it is a common experience of analysis that the Australian weather is usually characterized by a series of travelling anticyclones and by a tendency towards drought in the eastern regions, particularly in autumn and spring, when the maximum upper westerlies flow at higher latitudes along the polar edge of the continent or over the sea to the south.

It is concluded that hurricane seasons of maximum storm frequency over New England in America and over New South Wales in Australia, tend to be associated with relatively high pressures over the adjacent sub-tropical oceans, with a poleward ' steering ' along the coast and with strong, though perhaps intermittent upper westerlies at low latitudes. The same type of synoptic situation – even if it does not produce hurricanes – is conducive to intense and frequent rainfalls in the eastern Caribbean, the American east coast, the Australian east coast and also inland eastern Australia south of the Gulf of Carpentaria. These are all regions known to have been much wetter before the turn of the century than afterwards. It is inferred that this type of situtation occurred more frequently then.

A somewhat similar result can be deduced from Flohn's study which started from completely different premises. Flohn investigated the synoptic situation during recent severe winters. It appears that these were characterized by deep pressure troughs at the 500 mb level above tongues of cold air over eastern North America and over Europe. Warm air, carried north on the east side of these pressure troughs, can cause relatively heavy precipitation there. For example, in 1928-29 and again in 1939-40, severe winters over the North American plains were associated with above-the-average precipitation and relatively high temperatures all along the Atlantic seaboard, right into Newfoundland and thence across to Hudson's Bay. It would appear that cold cut-off lows, formed at the tip of the cold tongues above the interior of the continent, acted as centres that steered warm air and minor disturbances along the coast and towards the Bay.

Europe also had record cold during the same winters. This was associated in both cases with above-the-average rainfall along the North African coast, the Levant and in southern Persia; regions which Butzer and Kraus had shown to be sensitive to climatic variations. There was, however, no evidence of a steering of disturbances and warm air north towards Finland through eastern Europe. It is of some interest, although not necessarily significant, that the four winters (1928-29, 1939-40, 1941-42 and 1946-47) which were quoted by Flohn as having been particularly severe in recent times, were followed immediately by seasons of above-average rainfall at Mombasa, Nairobi and other East African rainfall stations.

Flohn's synoptic pattern for characteristically severe winters and Ballenzweig's pattern for hurricane seasons with maximum storm frequency are rather similar although they refer to different times of the year. Both are associated with strong upper westerlies at relatively low latitudes. In both cases there is a tendency towards poleward currents with heavy precipitation along the American east coast. Flohn explains the development as being due to a standing wave in the lee of the Rocky Mountains. I am more inclined to consider an intensification of the Atlantic high-pressure cell as the immediate cause of the development. It is there that energy is generated and that westerly momentum is being produced in the upper air. Also there are no high mountains in Western Australia. Although not impossible, it is difficult to explain the analogous development in the Australian region in terms of a standing wave. There is, however, no essential contradiction between Flohn's and my arguments. The two processes interact; either would tend to produce a moist southerly airstream along the Atlantic coast of America.

The relatively small amplitude of changes in the rainfall regime of South-east Asia can now be explained. On the one hand, periods which are generally wet in low latitudes and along the sub-tropical east coasts are probably associated with a relatively active direct meridional circulation and, therefore, presumably with strong upper westerlies at relatively low latitudes. On the other hand, it was shown by Riehl (1954) that the development of

he Indian summer monsoon is impeded as long as the upper westerlies flow south of the Himalayas. The reduction in the vigour of the circulation which appears to have occurred owards the end of last century reduced the amount of moisture available for transport owards the equatorial rainfall belt, but the associated poleward shift of the upper westerlies, avoured an early monsoon and possibly a deeper monsoon low. The influences of these wo oppositely changing effects upon rainfall in the Indian region appear to have balanced ach other fairly closely.

Over Siberia, during periods of active circulation, the strong upper westerlies to the outh should tend to shed cyclonic eddies which might disturb the establishment of trong and persistent anticyclones in winter. In summer, the upper westerlies would end to be more over Central Asia where their presence interferes with the development f a continental heat low. A general weakening of the upper westerlies and a poleward isplacement would reduce these effects and would favour a greater amplitude of the anuary-July pressure oscillation as was, in fact, observed by Kraus (1956). Strong wester- es in summer would also deflect maritime moisture-bearing winds from the north-eastern arts of Asia. This would be in good agreement with the studies of Yamamato (1958, ranslation) on the relationship between variations in the latitude of the jet-stream and ainfall over Japan.

Flohn thinks that the synoptic pattern which characterized recent severe winters as typical for a glacial age. The greater meridional gradient of surface temperatures in niddle and lower latitudes during glacial periods must have been associated with a more apid increase of mean westerly wind strength with height and with a displacement of the pper westerlies towards the equator. This is a direct consequence of the equations of notion. Also at present, the upper westerlies of the Northern Hemisphere are very nuch weakened during summer. During glacial periods they must have remained strong hroughout the year, as is still the case today in the Southern Hemisphere.

It is not certain whether there was a more active transport of heat, moisture and nomentum from low to high latitudes during the glacial periods. Although the meridional emperature gradient is stronger in the Southern than in the Northern Hemisphere at present, there is no evidence of a greater meridional heat flow. However, the strong neridional density and temperature gradient of the Southern Hemisphere can only be palanced by the centrifugal force of an upwards-increasing thermal wind, because frictional effects are relatively small over this ocean-covered part of the globe. In the Northern Hemisphere, with its continents and many mountain ranges, a balancing thermal wind is more readily disturbed and increased horizontal temperature gradients should be associated here almost inevitably with a more active meridional exchange of heat, moisture and other properties.

A more active meridional exchange is most likely to produce an intensified Hadley-ype circulation in the tropics. This would cause increased rainfall in the equatorial rough. In middle latitudes the exchange is largely effected by the type of meridional current found on the Ballenzweig and Flohn models along the American east coast. Eddies which are themselves being displaced in a meridional direction are particularly effective agents of exchange. For example, on the eastern side of polewards-moving cyclones, warm air can be carried through meridional distances far greater than the actual diameter of the depressions. During the last hundred years, colder periods at high latitudes occurred together with wetter conditions along the eastern seaboards of North America and Australia and vice versa. Much of the rain in those regions is due to poleward-moving depressions associated with the Ballenzweig type of situation. It can be inferred that the same type of situation, perhaps more intensely developed, was relatively frequent during glacial/pluvial periods.

According to Flohn, southerly moist airstreams – and the associated travelling disturbances – supplied the vast snow amounts necessary for the nourishment of the North American Keewatin and Labrador ice-sheets. Similarly the European ice-sheet, which had its centre east of the Scandinavian mountains, may have been nourished by precipitation from moist airstreams, which have been associated by Meinardus (1944) with

a more frequent occurrence of Vb-type cyclones. The earlier discussion indicated that the frequency of these Vb-cyclones and that of east coast cyclones in America may vary together. Over East Asia, the more zonal alignment of the mountains produces different circumstances. This caused recent climatic fluctuations there to have a different pattern and it may explain the small extent of the ice in that region during the last glacial period.

To summarize, glacial periods were probably characterized by a more active conversion of thermal into mechanical energy. An intensified Hadley-type circulation in the tropics caused heavier rainfall in the equatorial trough, and the associated increase in wind strength produced heavier evaporation from the oceans. The upper westerlies were also stronger, particularly in summer, and probably closer to the Equator. The actually dominant synoptic pattern is not known, though by analogy it has been inferred that deep meridional currents and poleward-moving depressions probably caused a vigorous exchange of heat and moisture between different latitudes particularly over eastern America and to a lesser extent also over Europe. The resulting northward transport of moisture may have helped to nourish the icefields there. It is concluded that Ice Ages were probably accompanied by pluvials in lower latitudes. This conclusion would also be in agreement with the world-wide pattern of climatic changes during the last hundred years.

6. The dynamic aspects of different climatic regimes

We now have two apparently necessary conditions for the establishment of a glacial period.

(a) The general temperature level, particularly in summer, must have been a few degrees lower. The amount of winter precipitation required to produce extensive glaciation in middle and low latitudes without such a temperature reduction is physically unlikely.

(b) Wetter conditions in the sub-tropics and greater meridional temperature contrasts must have been associated with a more active evaporation-precipitation cycle, and with a higher kinetic energy of the atmosphere as a whole.

On the average over the whole globe, regions where air is being heated must be at a higher pressure and therefore at a lower mean altitude than regions where cooling occurs. The generation of kinetic energy increases with both the mean potential temperature differences that exist in the circulating air, that is with the yield of the heat sources and sinks, and with the pressure differences between them.

The atmosphere is cooled mainly by the emission of infra-red radiation. It is heated by absorption, condensation and conduction or convection. All the heating processes depend to some extent upon the temperature of the Earth's surface, though the actual heating may occur at a higher level. The potential temperature increase at the heat sources must be equal to the decrease at the sinks when integrated over a sufficiently long time and over the whole globe. The radiational cooling exceeds, however, the radiational heating of the atmosphere by an amount approximately equal to 28 per cent of the total incoming solar radiation.

For the first time 30 years ago and again recently it was argued by Simpson (1957) that a decrease in solar radiation from its present volume could not bring on an Ice Age. A lower incoming radiation would reduce the radiational equilibrium temperature at the earth's surface near the Equator, resulting in a lower yield of some heat sources and hence in a reduced generation of kinetic energy. The establishment of glacial and of pluvial conditions require, according to Simpson, a higher rate of solar radiation than exists today. This would increase the potential temperature rise at the main heat sources and therefore the vigour of the circulation.

There are a number of difficulties in this theory. In particular there is no evidence that recent climatic fluctuations were associated with measurable changes in the amount of direct solar radiation reaching the earth's surface. The last interglacial should have been very wet according to Simpson but the evidence from Australia and from the fringes of the Sahara suggests that it was more arid than today. There also is no evidence, according to Brooks (1949), that the last interglacial was warmer than either the preceding one or

he postglacial period, as would be required by the theory. Finally, in order to account or the increased circulation, it has been suggested by Simpson that the surface air tempera- ures in all seasons at the Equator were 6°C higher than today at the maximum of a glacial hase. This is in contradiction to the oceanographic evidence which indicates substan- ially colder ocean surface temperatures in the tropics at that time.

These considerations make it difficult to accept Simpson's thesis of increased solar adiation during glacial and pluvial periods. However, they do not affect his basic theore- ical argument that during these periods the more active circulation must have been riven by a system of heat sources and sinks characterized by relatively great differences in otential temperature and possibly pressure. It is argued below that the establishment f glacial and pluvial conditions could be explained more plausibly by stipulating a more ntense radiational cooling from the free atmosphere, or possibly a lifting of the heat inks to a higher mean elevation. If that was associated with an unchanging intensity f the direct solar radiation at the bottom of the atmosphere, it would increase the generation f kinetic energy just as effectively as the process stipulated by Simpson In this case, owever, the more active circulation would be associated with a lower mean temperature f the air – a condition shown to be almost certainly essential for the establishment of a lacial phase.

More intense radiational cooling at high levels would tend to increase maritime tropical ainfall which already has a maximum at night. It would also tend to speed up the direct eridional circulation of the trades and hence intensify the sub-tropical high-pressure ells over the tropical oceans – a development which appears to be associated with wetter, ooler phases as discussed in the preceding section.

7. The heat sinks of the atmosphere

Cooling by radiation into space can be caused by any gas which emits and absorbs adiation and which has a mixing ratio which decreases with height. Water vapour atisfies these specifications. Radiational cooling caused by water vapour is diffused hroughout the middle and upper troposphere. It is concentrated particularly on the pper surface of clouds and haze layers where the mixing ratio tends to decrease discon- inuously. At present, the atmosphere between 30°S and 30°N gains on the average bout 23×10^{14} cal sec^{-1} by condensation and conduction of heat from the sea. More han 60 per cent of this amount, as well as all the radiational heat gains are being lost y infra-red cooling. The remainder is exported to higher latitudes to be lost there again ainly by long-wave radiation direct into space.

Cooling towards the ground is of a more local nature. It is most pronounced above ce or snowfields, and will tend to have dynamic effects if the cooling surface is at a higher levation than adjacent heat sources such as the open sea. The Antarctic ice-cap falls nto that category and so did the great continental ice-sheets of the Pleistocene.

The extensive cloud cover associated with a pluvial and the large snow-surfaces of a lacial period are both effective cooling agents and effective reflectors of sunlight. They an amplify many times over the initial cooling which caused their formation and are herefore self-perpetuating to some extent. For the case of ice this has been discussed in letail by Brooks (1949). The large-scale cooling effect of the clouds has been studied by impson and more recently by Wexler (Shapley 1953). However, both ice-sheets and louds are secondary phenomena. There still remains the problem of a primary distribu- ion of heat sinks which must have been associated with other factors, and which helped o establish in the first instance the peculiar distribution of water in all its phases in the tmosphere and on the ground which characterizes a glacial or pluvial age.

Water vapour is transported upwards from the Earth's surface by convection. Radia- ive cooling from high, humid layers and from cloud tops then helps to maintain this onvection. The tropopause marks the upper boundary of the region in which convection s dominant. Above it, in the stratosphere, radiative equilibrium tends to prevail.

As discussed by Goody (1949), the equilibrium temperature and height of the tropopause depends on the surface temperature and on the relative abundance of water vapour, carbon dioxide and ozone. At middle and high latitudes, ozone has little effect. The equilibrium level and temperature of the tropopause there depends mainly upon a balance between radiative heating caused by CO_2 – the mixing ratio of which is constant with height – and cooling caused by water vapour. Conditions are different over the tropics. The radiation due to CO_2 and H_2O is small at the tropical tropopause, because the cold, tenuous air can only contain a small mass of these gases. The ozone amount in the tropical stratosphere, though also smaller than in higher latitudes, is not reduced in the same large ratio. It has therefore been suggested by Goody that the effect of ozone – and of its variations – is probably more important in the tropics than at higher latitudes. A decrease of the stratospheric ozone mixing ratio would tend to lower the temperature and raise the height of the tropical tropopause. According to Plass (1956a) it would also tend to lower the equilibrium temperature at the top of clouds and hence intensify the cooling rate within the troposphere.

A higher tropopause is equivalent to a deeper convective layer. From geometric and dynamic reasons, discussed by Bjerknes (1937) and Høiland (1939) it can be deduced that this would tend to increase the extent and the intensity of convective disturbances as well as speed up the direct meridional circulation. It will be seen that a reduction in the ozone amount of the tropical stratosphere might cause higher rainfall and possibly a pluvial regime. The more intensive extraction of heat from the troposphere would also reduce the amount of heat available for export to extra-tropical latitude, and this would be conducive to colder conditions there.

On the other hand, an increased generation of ozone, particularly if it occurred at comparatively low altitudes, would tend to decrease the thickness and cooling rate of the tropical troposphere. This would be associated with less extensive convection, a less active circulation and a reduced poleward transport of heat – conditions which are closer to those of a non-pluvial, interglacial regime.

Variations in the ozone content might possibly be the cause of the relatively short climatic fluctuations with minor amplitude. It seems rather more doubtful whether they could account by themselves for the sequence of glacial and interglacial stages. Finally it is most unlikely that they could be used to explain the possible existence of moist and warm climates in the past or the persistence of high temperatures in polar regions. The character of these past climatic regimes might possibly be explained by a CO_2 mixing ratio substantially different from that prevailing today.

A low CO_2 mixing ratio would cause relatively cold surface temperatures at high latitudes. In the same way as a low O_3 mixing ratio, it would also cause a high tropopause, a more intense cooling from vapour layers and cloud tops, and hence heavier precipitation in the tropics. On the other hand, a moderately increased CO_2 mixing ratio would result in reduced meridional contrasts and in a warmer, more arid climate, as occurs during non-glacial interpluvial periods.

A climatic regime of this type is only quantitatively, not fundamentally, different from the present. A still greater increase of the CO_2 mixing ratio might produce a radically different picture. The tropopause would be much closer to the ground. The vertical stability of the greater part of the atmosphere would then tend to suppress vertical motion almost entirely in favour of horizontal movement. There may be no rain because the convective troposphere would be too shallow for raindrops to form by known processes. Small-scale turbulence and molecular conduction would tend to establish some equilibrium in water pressure between the sea surface and the atmosphere. The result would be a hot, humid and possibly always foggy climate. We do not know whether the CO_2 mixing ratio was ever sufficiently large to produce these conditions, though it is not impossible that they are being approached on Venus today. They are so different from our present synoptic meteorological experience, that the latter ceases to act as a guide.

8. Primary causes and discussion

On the basis of the preceding argument the whole pattern of climatic fluctuations could be explained by changes in the infra-red cooling rate. These could be due to changes in either or both the O_3 and the CO_2 mixing ratio. Although this explanation is apparently consistent with observable events, it cannot be proved to be the true explanation. However, it is difficult to find any other factor which could affect world climate, which has not itself remained invariant during the most recent climatic fluctuation and which would produce the type of change which did in fact occur.

In particular, the astronomical elements of the Earth's motion, the tidal force, and the configuration of the Earth's surface have not changed recently; nor has probably the direct solar radiation at the Earth's surface. The dust content of the atmosphere may have changed. Wexler (Shapley 1953) suggests that it has decreased at the beginning of this century and that this caused increased solar radiation and a gradual warming at the earth's surface. However, if true, this would also cause a more active circulation which is contrary to the actual pattern of events. This pattern could not be explained either by the nucleating properties of dust and their effect on ice-crystal and rain formation. This effect would be least important in low-latitude, maritime regions where rain forms mainly by the coalescence process, but very significant changes in rainfall occurred just there.

On the other hand, changes extending over several years or decades, in the tropical stratospheric ozone content, though not recorded, appear to be not altogether implausible. Ozone is generated under the influence of short-wave ultra-violet radiation in the Schumann-Runge and Herzberg bands. It is destroyed mainly by ultra-violet radiation of somewhat large wavelength in the Hartley band which has a maximum near 2500 Å. The square root of the ratio in the intensities of these two radiations, determines the photo-chemical equilibrium amount of ozone, which may be found under given pressure and temperature conditions. Most of the ozone in middle and high latitudes has been transported there by horizontal air currents from lower latitudes and it is not in photo-chemical equilibrium. Near the Equator, equilibrium conditions are more closely approached. A higher ratio of Schumann-Runge to Hartley radiation might cause there an increase in total ozone amount. Because of the relatively deeper penetration of the ozone producing radiation in this case, it might also cause a lowering of the ozone layer at low latitudes. This view would be in agreement with observations discussed by Karandikar and Ramanathan (1949), who found that the height of maximum ozone content over Delhi dropped from 30 km when the total amount was 0·155 cm, to about 22 km when the total amount increased to 0·217 cm. High total ozone amounts, particularly at relatively low altitude, would cause a lower equilibrium height of the tropical tropopause and reduce infra-red cooling in the upper troposphere.

There is a considerable amount of circumstantial evidence to show that solar radiation is not invariant in the ultra-violet. As the sun radiates apparently at constant rate in the visible part of the spectrum, the variability – if it does exist – must be different for various wavelengths. However, whether comparatively long-term fluctuations do in fact occur, is completely unknown at present.

The CO_2 mixing ratio appears to have increased during the last century (Callendar 1958). Natural self-sustained oscillations of atmospheric CO_2 have been discussed by Erikson and Welander (1956). Plass (1956b) has shown that cyclical changes with a period of some 10,000 years may occur as a result of interaction between air and ocean. Variations may also be caused by the fixation of CO_2 through marine organisms, by volcanic eruptions and by weathering.

The professed purpose of this study was to interpret all known climatic fluctuations with a minimum number of assumptions. It may be of some interest to enumerate again some of the assumptions that have been made, as a possible guide to future research. I have tried to relate observed climatic phenomena to unknown variations in ultra-violet radiation and CO_2 content. More could be learned about both, through long series of observations with now available techniques. The same applies to variations in the tropical

O_3 mixing ratio and their relation to the sun. The effect of both CO_2 and O_3 upon the tropospheric cooling rate and upon the height of the tropopause has been studied by a number of competent investigators, but there still appears to be scope for quantitative refinement and confirmation. Not enough is known about the relation between the circulation pattern and the distribution of heating and cooling. I have used circumstantial evidence to deduce the prevalence of certain synoptic situations in the past. The result is conjectural and very uncertain but it is not essential to the present argument. It could be tested by theoretical and synoptic studies of the general circulation, or by the investigation of fossil dunes and other geological phenomena. Finally, the probable coincidence of pluvials near the Equator with glacial conditions in higher latitudes, which has been inferred on the basis of physical considerations and synoptic analogues, cannot be accepted as proven until it has been confirmed by additional geological evidence.

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Weather in wheat: an essay in micro-meteorology

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SUMMARY

Data for three summers (1955-1957) are presented, obtained from continuous recorders responding to temperature, humidity and wind in and above the wheat crop at levels up to about 2 m. Section 1 gives a brief account of the instruments (see Long, *Q.J.* **83**, p. 202 for details). § 2 a description of the site, 70 × 70 m, which may have been too small for full development of profiles. § 3 (from 1955) shows that planting density affects the daily cycles of temperature and humidity in the crop, a thin crop having a lower average relative humidity and a shorter period of saturation on a dew night. Such a night is characterized by a humidity lapse between ground and canopy, and a humidity inversion above the canopy. On the exceptional occasion illustrated, both the up flux and the down flux may have reached a maximum rate of about 3 mg ground cm^{-2} hr^{-1}. An adventitious air-temperature ripple during a calm night is exploited to derive rough estimates of transport constant in a dense crop: near the ground it approached the molecular thermal diffusivity for air. § 4 (from 1956) displays anomalies in temperature and humidity profiles that can arise from interpolating readings within the 400 sec cycle of the recorders. It also gives soil temperature records from which estimates of soil thermal diffusivity, and soil heat flux, are made, indicating that on the first sunny June day after rain there may be a net heat flux into the ground of about 30 cal cm^{-2}: the equivalent of 0·5 mm of evaporation. From an energy balance further rough estimates of heat transport constant within the crop are found to be of order 2 to 20 cm^2 sec^{-1} depending on height and external wind speed. The anomalies in profiles are attributed partly to periodicities in the air parameters almost identical with that of the recording instruments, and partly (§ 5, from 1957) to long-persisting differences in temperature and humidity at constant level across the site. Within the crop an average temperature difference of 0·8°C may persist for four hours at positions 8 m apart (and a vapour-pressure difference of 1·0 mb), and these 'hot' and 'damp' spots – not always coincident – move slowly during the day. The differences are smaller above the crop where mixing is more thorough. These differences swamp any that might be due to advective effects, and raise doubts about the meaning of 'a profile' over a tall crop.

§ 6, introduces an analytical Part II which uses the data from a period of eight days (12-19 June 1957), of fine weather after rain, discusses the generalized profile equation

$$u = \frac{u_*}{k\alpha}\left\{\left(\frac{z-d}{z_0}\right)^{\alpha} - 1\right\},$$

(reducing to the standard logarithmic profile when $\alpha = 0$): measurements of temperature and humidity are not accurate enough to determine α for their profiles, and the swaying of the crop necessitates use of long-period averages of wind speed before α can be determined for wind profiles. Working on 4-hr periods (00-04, 04-08 and so on) it is found (§ 7) that profiles above the crop can be fitted either with $\alpha = 0$, or a value of α chosen from the Deacon curve of $1 - \alpha$ against Richardson number. Values for a crop height of about 60 cm, with heat transfer (Q_s) and evaporation rates (E) from later sections, are:

Period	0-4	4-8	8-12	12-16	16-20	20-24
Ri	0·080	− 0·007	− 0·017	− 0·010	0·014	0·046
Chosen α	0·20	− 0·03	− 0·05	− 0·04	0·03	0·10
u_*/k cm sec^{-1}	17	75	158	181	127	66
u_*/k cm sec^{-1} for $\alpha = 0$	59	67	132	158	130	92
z_0 cm for $\alpha = 0$	5	5	11	14	12	12
d cm for $\alpha = 0$	38	37	22	14	18	25
Q_s (calc.) cal cm^{-2}/4 hr	− 15	+ 4	+ 32	+ 14	− 26	− 33
E_A (calc.) cal cm^{-2}/4 hr	− 0·2	10	82	106	49	14
E_B (meas.) cal cm^{-2}/4 hr	− 0·2	24	96	107	57	5

The discrepancy in u_*/k during the first period corresponds to a 12-fold difference in estimated shearing stress.

Wind profiles in the crop have a completely different shape in calm and windy weather, in the sense that near the ground the absolute wind speed tends to be constant: the crop behaves as though self-sealing.

Temperature and humidity profiles (§ 8) plotted in the form of wind speed against T or e at the same height are, statistically, straight lines above crop level, and in § 9 these straight lines are used to determine sensible heat and latent heat transfers (using $\alpha = 0$) for the 48 4-hr periods in the 8 days. The daily totals, and the mean diurnal cycle of evaporation agree well with direct measures taken simultaneously. Evaporation at night is negligible (see previous table). The daily cycle of temperature and humidity (§ 10), considered in a little more detail during transitions to inversion or lapse, on a ' normal ' day and on a day of anticyclonic subsidence, indicates that the presence of very dry air has not increased evaporation rates : the local energy balance is still dominant. It is suggested that the crop has met this threat of potentially severe desiccation by stomatal closure, and the speculative § 11 attempts to calculate rates of transfer from the leaves into the ambient air, at various levels in the crop, in terms of leaf morphology, leaf temperature, and a ventilation factor. Applied to two periods, one calm with wet-surface soil, the other windy with dry-surface soil, it appears that on the first occasion about half of the evaporated water came from the soil, and on the second occasion, none. The total fluxes so estimated agree well with aerodynamic estimates based on wind and humidity profiles over the crop. Derived estimates of transport constant, well down in the crop, show near identity in calm and in windy periods (cf. wind velocity measurements § 7), and at the crop surface they agree well with aerodynamic estimates. A short § 12 discusses some biological and physical implications of the results.

1. Introduction

During the past ten summers measurements have been made of the physical environment of farm crops, mainly as vertical profiles, but more recently some measurements have been made at constant level across the test site. A short account of work on potatoes (Hirst, Long and Penman 1954), a very brief reference to miscellaneous crops (Long 1958), and a detailed account of instrumentation (Long 1957) have been published. Most of the many data accumulated during the summers of 1955, 1956 and 1957 consist of temperature readings (wet-bulb and dry-bulb) and wind-velocity readings, and it is on these data that most of this paper is based. The nickel resistance thermometers were made to work into Kent Multelec 12-point recorders having a range 0-50°C (on 25 cm width paper) for 18 ohms change in resistance. It is easy to read temperatures to the nearest 0·10°C and this has been normal practice, but reading to 0·05°C is possible, and close to saturation, even smaller differences between dry- and wet-bulb temperatures can be detected. In his account of the instruments Long claimed that a temperature read from the chart was within 0·05°C of the temperature of the sensing element, and apart from a few special cases with special reasons, severe probing of the three years' data has shown the claim to be fair. What was, and still is, doubtful is the agreement between the temperature of the sensing element and that of the ambient air to which it is responding, and discussion of this point will take its place in its appropriate context. Wind speeds have been read from continuous records, obtained from a step-by-step integrator activated by contacts on modified Sheppard-type sensitive-cup anemometers. On these a contact is made for every 38 rotations of the cups ($\simeq$ 62·5 m run-of-the-wind) and full-scale deflexion on the chart, corresponding to 100 steps, is 7·5 cm. With a time scale of 1·15 cm to the hour it is estimated that moderate to high wind speeds can be read to within $\pm$ 2 cm sec^{-1} over an hour. At low wind speeds the limitation on accuracy is that imposed by the instrument itself.

There has been conscious bias in the selection of material for this survey, most of the data being for fine summer periods free from rain. The choice of days with large ranges in temperature and humidity offers a better chance of displaying the inter-relations of the physical parameters in the crop environment, provides the least unreliable information about profiles from which calculations of transport rates can be made, and puts the greatest stress on the adequacy of the equipment.

Part I. *Chronological and qualitative*

2. The site

The area available has always been recognized as being too small. It is just over one acre (roughly 70 × 70 m), giving a maximum fetch to the middle of about 35 to 50 m according to wind direction. In the middle is the large balance on which continuous

records of transpiration are made, and the masts carrying equipment are set up close to the balance. The small area made imperative what was already desirable, namely, some study of the variations in temperature and humidity at constant level across the site. This was done in 1957, and is reported in § 5. The edges of the area run N-S and E-W. To the north and west is the remainder of an 11-acre field, giving a good open exposure except on a line close to north-west where a small clump of tall trees 110 m away has occasionally been suspected of affecting airflow over the site in a NW. wind. To the south, beyond an open fence, lies another 11-acre arable field, and then a long avenue of trees running east to west 200 m from the site. To the east is the meteorological enclosure, under grass and instrument shelters, about 20 m across, and then there is a large open field of 10 acres which carries a root crop every year, and it is usually the end of June before anything like a complete green cover is established. Earlier than this there is much bare ground, and as some of the fine early summer weather has come to the site on a NE. wind it has occasionally been suspected that the air is warmed in its passage across this particular field. Exposure to the south-east is very bad; but nothing in this paper refers to periods when the wind was in the quadrant E-S.

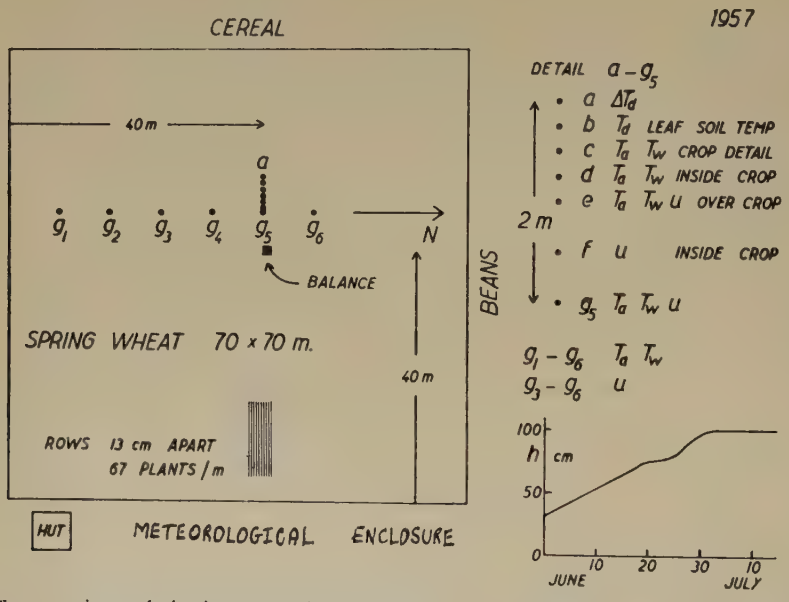


Figure 1. The experimental site in 1957, with a diagram of crop height during the period considered in the paper. In 1956 sites (d) and (e) were 3 m apart; in 1955 the wheat was on the north side of site.

Fig. 1 shows the arrangement in 1957 from which most of the records to be discussed quantitatively were taken. The layout was simpler in 1956, and very simple indeed in 1955, but the essentials common to all three years are the items labelled (d) and (e). Pairs of thermometers (wet and dry) were set up within the crop, between ground level and the top (d); and on another mast a short distance away other pairs were set up above the crop, and, at the same levels, cup anemometers (e). In 1956 and 1957 the levels were adjusted daily as the crop grew so that positions with respect to the top of the crop remained constant.

3. Results for 1955

(i) *Daily cycles of temperature and humidity*

The main work in 1955 was on potatoes, but as the rest of the field was under spring wheat some observations were taken there to exploit an accident by which part of the area had been drilled twice to give a very dense plant cover. Part of the rest of the normal

crop was hand-pulled, leaving only one plant in four. Naming the three areas (a) dense, (b) normal, and (c) thin, the distribution of equipment was :

Pairs of thermometers	(a)	2·5, 5, 10, 15, 20, 30 cm
	(b)	12·5, 25, 100, 200 cm
	(c)	12·5, 25 cm
Anemometers	(b)	12·5, 25, 100, 200 cm
	(c)	12·5, 25 cm

All heights are measured from ground level, and as on the 4-5 June the crop heights were (a) 31 cm, (b) 22 cm, (c) 18 cm, *all* of the (a) thermometers were inside the crop, and the lowest anemometers on (b) and (c) plots were inside the crop. This technique of measuring wind speed inside the crop by cup anemometers was a failure, as the need to clear an area round them, and the inevitability of an open space above, made any quantitative use of the data very uncertain.

Comparisons at different crop densities yielded little that was unexpected. Table 1 gives a brief statement of temperature and humidities at epochs during the period 4 and 5 June, two days separated by a clear calm night when dew was formed. Discussion is difficult because crop densities and crop heights are different, but it appears that by night (periods A, L and N) the thin crop is the warmest at 12·5 cm, whereas about midday (periods E and U) the dense crop is the warmest. There are similar changes-over in the vapour pressure, but the compound picture is probably best seen in the values of relative humidity at 12·5 cm, given on the left of the vapour pressure readings. The thin crop is

TABLE 1. TEMPERATURE AND HUMIDITY 4-5 JUNE 1955

Day	Period	Height cm	Temperature °C 12·5	25	100	200	R.H. (per cent)	Vapour pressure (mb) 12·5	25	100	200
		(a)*	11·74	11·97			†(87)	12·00	11·67		
4	A 00-02	(b)	11·92	12·22	12·60	12·68	(83)	11·48	11·14	10·74	10·50
		(c)	12·24	12·34			(79)	11·22	11·08		
		(a)	10·29	10·20			(100)	12·52	12·42		
	C 05-07	(b)	10·10	10·08	10·08	10·02	(100)	12·36	12·30	12·04	11·96
		(c)	10·08	10·00			(100)	12·32	12·26		
		(a)	15·13	15·23			(98)	17·05	16·22		
	E 10-12	(b)	14·88	15·02	14·52	14·58	(94)	15·90	14·76	13·98	13·66
		(c)	15·04	14·84			(89)	15·06	14·88		
		(a)	16·46	16·81			(85)	16·00	14·50		
	H 15-17	(b)	17·16	16·68	15·94	16·06	(70)	13·70	12·68	11·54	10·58
		(c)	17·12	16·62			(68)	13·28	12·30		
		(a)	6·05	5·51			(100)	9·30	8·94		
	L 22·30-24	(b)	6·15	6·45	7·80	8·05	(100)	9·40	9·53	9·67	9·77
		(c)	6·73	6·73			(98)	9·63	9·75		
		(a)	3·99	3·94			(99)	8·03	8·09		
5	N 02·30-04·30	(b)	4·18	5·08	6·50	6·88	(100)	8·24	8·48	9·60	9·80
		(c)	5·56	5·68			(97)	8·78	8·96		
		(a)	17·11	17·94			(84)	16·38	15·30		
	S 07·30-09·30	(b)	17·22	16·28	15·02	14·78	(78)	15·28	13·04	11·64	11·04
		(c)	16·04	15·92			(75)	13·66	12·62		
		(a)	20·05	20·20			(75)	17·62	15·90		
	U 11-12·30	(b)	19·65	18·40	17·42	17·20	(69)	15·75	13·10	11·57	11·42
		(c)	18·55	18·50			(66)	14·00	13·43		
		(a)	14·96	15·38			(86)	14·64	14·02		
	X 18-19	(b)	15·67	15·67	15·67	15·77	(78)	13·93	13·27	12·80	12·30
		(c)	15·73	15·83			(75)	13·27	13·27		

* All (a) values are means of those at 10 and 15, and at 20 and 30 cm. † Values in brackets are relative humidity (per cent) at 12·5 cm.

the driest except when all are saturated, and from the Table can be inferred what is obvious from the complete record, that the thinner the crop the lower the mean daily relative humidity, and the shorter the period for which it remains at, or near, saturation. As fungus behaviour is determined by relative humidity rather than by absolute humidity this aspect of the records will be pursued further elsewhere.

The major physical interest is in the detailed profiles within the crop, obtained on the dense plot. Although these are probably not typical of normal spring wheat, they may be representative of winter wheat (which has a much greater leaf development) and possibly of other tall plants that acquire a close canopy completely shading the ground. Fig. 2 presents a two-day record of weather in two ways. The upper half gives values of temperature, wind speed and vapour pressure at hourly intervals : the lower half gives temperature and humidity profiles at a few selected epochs. It is assumed that readings made over area (b) at 100 cm and 200 cm are equally representative of area (a), and as the discussion is to be almost entirely qualitative there seems to be little risk of serious misinterpretation. The first set of three curves shows the temperatures near the ground (5 cm), just inside the top of the crop (30 cm) and well clear of the crop (200 cm), reveals the major

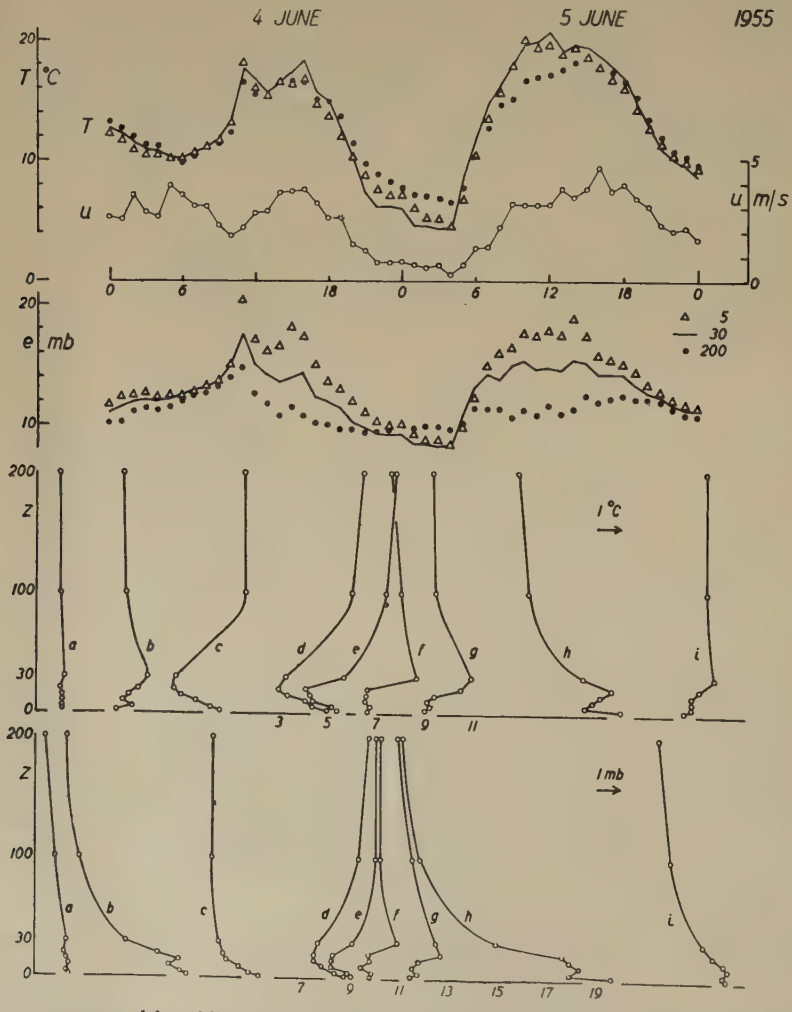


Figure 2. Temperature and humidity for a dense crop on a dew night. Top : hourly values 4-5 June, at stated height (cm), with wind speed at 200 cm. Bottom : T and e profiles : (a) mean 05-06, (b) mean 12-13, (c) at 2130 on 4 June, (d) at 04, (e) at 0430, (f) at 05, (g) at 0530, (h) mean 10-11, (i) at 18 on 5 June. Crop height 30 + cm.

hanges from lapse to inversion, and the great cooling on the clear calm night in the middle f the period. The wind speeds at 200 cm are plotted immediately below. The correspond- ng vapour-pressure curves form the next group of three, and it is to be noted how both apour pressure and temperature decrease before dawn and then increase very rapidly rom sunrise onward, but the other features of the record are better seen in the profiles. The first (a) (5-6) is the mean for the hour, and is for the period of minimum temperature n the morning of 4 June. There is zero temperature gradient, but there is a slight vapour- ressure gradient, representing a small upward flux of vapour from the ground and through he crop. The second (b) and eighth (h) are characteristic daytime profiles with the top f the crop as the hottest part, small temperature gradients above the crop, and marked apour-pressure gradients, as expected. The third to seventh (c)-(g) are best considered as group, taking vapour pressure first. By 2130 hr (c) the vapour-pressure gradient above he crop had fallen to zero, and from then until sunrise (curve (d), for 04 hr) was inverted, ndicating downward transfer of vapour from air to crop surface. During the whole of his period the lapse was maintained in the crop, indicating upward transfer of vapour rom soil surface to the top of the crop, i.e., from both sides the canopy was accumulating vater. In the same period the air at this level was steadily getting drier (see time sequence reviously), and conservation principles were satisfied by condensation : the period was ne of intense dew formation. This group of profiles, and that for the hour 10-11 (h) re all on the same scale, i.e., the progressive movement to the right represents increasing umidity of the air. Within $1\frac{1}{2}$ hr the water content of the air in the crop increased by bout 50 per cent as the dew began to evaporate, and at the end of this interval the profile was almost the mirror image of what it had been at 04 hr. Unexpectedly, but on reflection ot surprisingly, some of this evaporated water was returning to the soil.

At 04 hr the vapour-pressure gradient above the soil was about -1 mb/10 cm, and above the crop about $+1$ mb/20 cm. Anticipating later conclusions, the transport constants in these calm conditions might be as much as 10 and 20 $cm^2\ sec^{-1}$ respectively, indicating equal transport rates, upward and downward, of about 3 mg $cm^{-2}\ hr^{-1}$. This is about the upper limit of rates measured or inferred by Monteith (1957) over grass, but the night of 4-5 June 1955 was one of exceptionally intense dew formation.

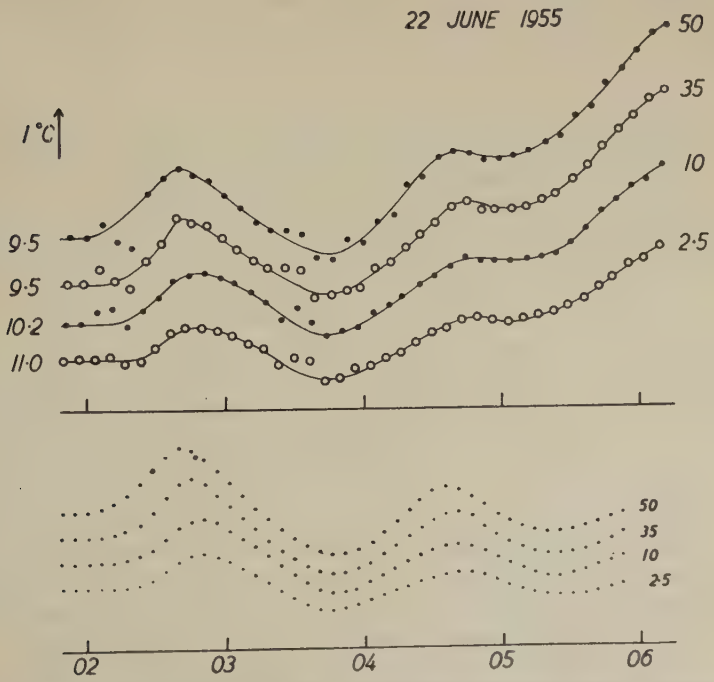


Figure 3. Temperature ripple 22 June 1955. Observation heights on right. Crop height 61 cm.

(ii) A temperature ripple, and an estimate of transport constant in the crop

On occasions there has been evidence of a short-lived periodicity in the air temperatures, a clear example starting just before sunrise on 22 June 1955 when the crop height was 61 cm. Ripples appeared on the previously smooth temperature traces and persisted for about four hours as the normal morning rise took place. Fig. 3 shows the phenomenon plotted on an open time scale, and, as objectively as possible, smooth curves have been drawn through the points, and through the long-period means. The departure of the smooth curve from the mean is plotted in the lower half of the diagram, for the levels 50, 35, 10 and 2·5 cm, and there is evidence of two cycles with approximately equal periods of two hours, and of the decay and delay of the wave as it moved downward. The curves give some information about the thermal diffusivity of the air in the crop, but the answers from alternative methods of estimating the constant are too widely different to place much faith in either. The values are in Table 2, with the third column based on the expression given by McCulloch and Penman (1956) which combines the amplitude ratio and phase lag to get a single value of diffusivity.

TABLE 2. DIFFUSIVITY IN WHEAT 22 JUNE 1955

Height range cm	K cm^2 sec^{-1} From amplitudes	From phases	From phase and amplitude
50-35	6	1·3	1·0
35-10	3	5	4·6
10- 2·5	0·3	7	2·8

Wind speeds at 200 cm ranged from about 60 cm sec^{-1} to 100 cm sec^{-1} during the period, and the air inside this dense crop must have been nearly motionless. Very tentatively, it appears that the transport constant is of the order of a few cm^2 per second; even more tentatively, there is a hint in column 1 that close to the ground it might be of the same order as the molecular diffusion coefficient.

4. RESULTS FOR 1956

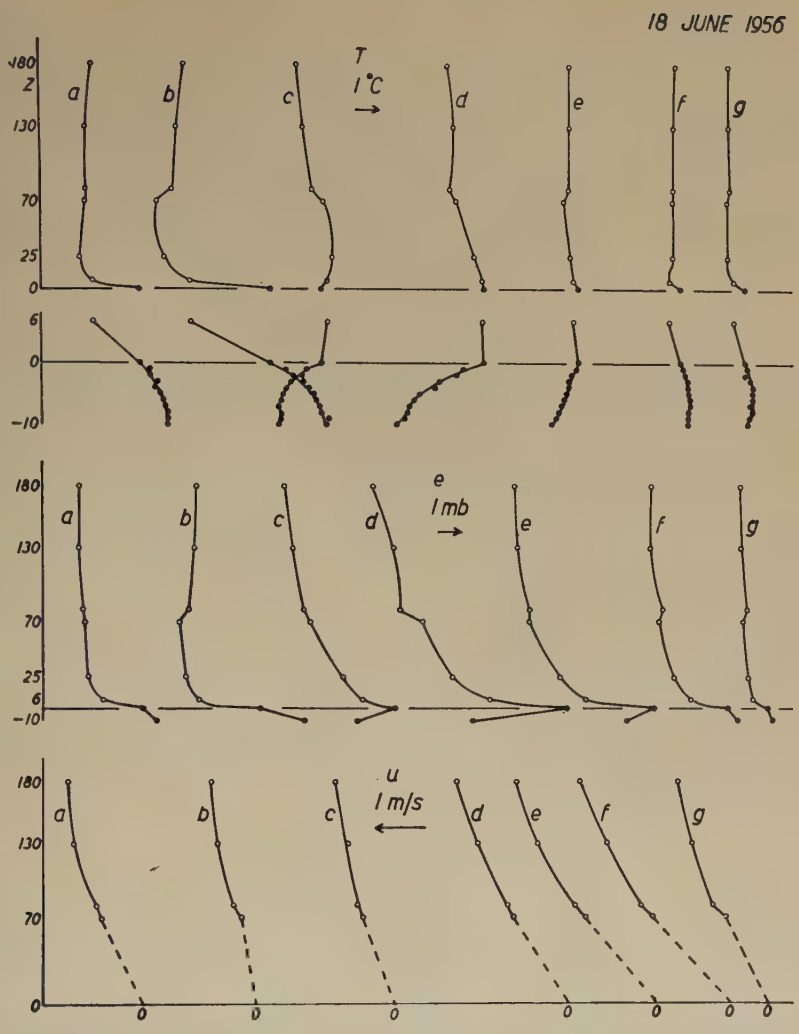
In 1956 and 1957 it was thought desirable to acquire more information above the crop and the standard profile was obtained from measurements at six levels, using the height of the crop h as a reference level and, in the active growing season, making daily changes in the upper four levels with less frequent changes in the lower levels. Pairs of thermometers (wet and dry) were set at

$$z \simeq h/4,\ h/2, \text{ and } z = h \text{ on one mast}$$

$$z = h + 10,\ h + 60,\ h + 110 \text{ on a second mast.}$$

In 1956 these masts were about 3 metres apart. For a period of a few weeks, thermometers intended for another experiment were given a trial run under the wheat, being set at 1 cm intervals of depth from $z = 0$ to $z = -10$ cm. Cup anemometers were run at $z = h$, $h + 10$, $h + 60$, and $h + 110$.

The main example from 1956 results is taken from this period as it is the only one in the three years that has fine detail of soil temperature changes to go with it. Apart from this, the chief use of the example is to expose a technical difficulty in processing the data, and the main discussion of the physics of the profiles will be left until the 1957 data are reviewed. In the hope of getting a quick transformation of dots on a chart into a useful diagram, the wet- and dry-bulb temperatures were read from the charts at hourly intervals by interpolating between successive values that straddled the hour line on the



gure 4. Air temperature, soil temperature (on 5-times scale), vapour pressure and wind at (a) 00, (b) 04, (c) 08, (d) 12, (e) 16, (f) 20, (g) 24 hr, 18 June 1956. Crop height 70-cm.

art. The result of such treatment appears in Fig. 4 for 18 June 1956, where only seven ts of values are given at 4-hr intervals, a convenient division of the day when the sun ses about 04 hr and sets about 20 hr. The four parts of the figure show : dry-bulb mperature; soil temperatures on a more open scale, together with the lowest level air mperature; vapour pressure, with soil values, assuming 100 per cent relative humidity at e soil surface (the ground was known to be wet) and at 10 cm below (fairly certain, within per cent, even if the surface is dry); and wind speed, plotted from right to left.

The technical difficulty is shown most clearly in the vapour-pressure profiles. That r 12 hr (curve d) shows an exaggerated lapse over the 10 cm immediately above the op, regarded as being improbable; profiles (f) and (g) show distinct inversions of humidity, garded as impossible. These are not momentary deviations from what was expected, ught here by an accident of sampling, but are representative of periods. To a smaller tent the same effect occurs in the air-temperature profiles, and it is worth noting that y correction of air temperature that smooths these profiles will *increase* the anomalies the vapour-pressure profiles unless at the same time a bigger correction is applied to e wet-bulb temperature. As this degree of adjustment is much more than the confidence mit in the measurements, the explanation of the anomalies as caused by instrumental

inadequacy has been set aside as improbable, though it cannot be completely rejecte Four possibilities have been considered : (i) differential radiation errors (set aside becau the effect occurs in darkness also); (ii) small local convective effects (set aside becau there is no clear association with temperature gradients in the crop); (iii) fortuitous u favourable phase relationships in the recording at the two levels (considered later in th section); and (iv) the existence of differences in temperature and of humidity, at consta level, across the test site (considered in § 5). Before looking at these sources of uncertain it is convenient to examine the information supplied by the soil-temperature records.

(i) *Heat exchanges between crop and soil*

The soil-temperature records give some information about the diurnal heat flux on fine day after a dull period. During dark periods when heat flow is upward out of the s the temperature profile shows no obvious discontinuity in passing through the surfac and for periods of a few hours (see for example, the last two (f) and (g) in the detaile soil profiles) the profiles are very nearly parallel, i.e., the heat flux remains constant, condition assumed by Brunt (1944, p. 139) in his treatment of night cooling under cle skies. Using the 11 soil-temperature records and plotting the diurnal cycle of each it ha been possible to get estimates of : (1) the variation of thermal diffusivity with depth, an assuming a value of specific heat per unit volume, ρc, to derive the corresponding variatio of thermal conductivity; (2) the heat flux at $z = 0$, and $z = -10$ cm; (3) the net chan in heat content between $z = 0$ and $z = -10$ (again assuming ρc is known); (4) a che on the balance between (2) and (3). Full details would need a separate paper, but briefl the diffusivity increases from $4{\cdot}2 \times 10^{-3}$ cm^2 sec^{-1} at the surface to $9{\cdot}6 \times 10^{-3}$ cm^2 sec at 10 cm. Over the 24 hr the mean temperature gradients were 0·134°C cm^{-1} at the surfac and 0·046°C cm^{-1} at 10 cm, and the average rise in temperature in the period was 1·65°C Hence the net downward flux of heat in 24 hr was :

	$\rho c \times 49$	cal cm^{-2} at $z = 0$
	$\rho c \times 37$	cal cm^{-2} at $z = -10$
Net gain	$= 12\,\rho c$	cal cm^{-2} between $z = 0$ and $z = -10$ cm
Measured gain	$= 10 \times 1{\cdot}65\,\rho c$	cal cm^{-2}
	$= 16{\cdot}5\,\rho c$	cal cm^{-2} between $z = 0$ and $z = -10$ cm

The near agreement in the two estimates is fortuitous, for the estimates of the diffusivit are not good enough to give much precision, and the agreement also depends on th identity of three values of ρc (surface, 10 cm, and mean). Fortunately this is a fairl conservative quantity in a soil profile as wet as this, and taking a value of 0·6 (known fro other measurements to be acceptable) the fluxes become 29 and 22 cal cm^{-2} day^{-1} an the net gain in the top 7 cm layer 10 cal cm^{-2} day^{-1}. This net down flux of 29 cal cm on a fine day after a dull period is in accord with a measurement of Monteith (1958) wh found 27 cal cm^{-2} day^{-1} under wheat using heat-flux plates. The amount is equivalent t 0·5 mm of evaporation, and is between 10 and 20 per cent of the transpiration to be expecte on such a day.

If complete radiative equilibrium can be assumed between the ground surface an the leaf canopy at night, then a rough estimate (very rough, and an overestimate) o transport constant in the air can be made for the region just above the soil surface. Th balance equation is

$$G = R + Q_s + Q_L$$

where

G is the sensible-heat flux from the ground

R is the net-radiation transfer to the canopy

Q_s is the sensible-heat transfer to the air

Q_L is the latent-heat transfer to the air

the working temperatures, to an adequate degree of accuracy

$$R = 0{\cdot}5\ (T_0 - T_{70}) \quad \text{cal cm}^{-2}\ \text{hr}^{-1}$$
$$Q_s = -1{\cdot}08\ K\,\partial T/\partial z \text{ (air) cal cm}^{-2}\ \text{hr}^{-1}$$
$$Q_L = -1{\cdot}64\ K\,\partial e/\partial z \text{ (air) cal cm}^{-2}\ \text{hr}^{-1}$$
$$G = -9{\cdot}1\ \partial T/\partial z \text{ (soil) cal cm}^{-2}\ \text{hr}^{-1}$$

s the night conditions were at or near 100 per cent relative humidity, from the slope of e saturation vapour-pressure curve it is possible to combine Q_s and Q_L to obtain

$$Q_s + Q_L = -2{\cdot}5\ K\,\partial T/\partial z \text{ (air) cal cm}^{-2}\ \text{hr}^{-1}$$

d

$$K = -0{\cdot}4\ (G - R)/(\partial T/\partial z).$$

he data and derivations for the periods 0-04 hr and 20-24 hr on 18 June 1956 are shown Table 3.

TABLE 3. NIGHT-TIME HEAT BALANCE UNDER WHEAT 18 JUNE 1956

me	$T_0 - T_{70}$	$T_0 - T_6$	$-\frac{\partial T}{\partial z}$ (soil)	R	G	$G - R$	$(T_0 - T_6)/6{\cdot}25$	K
	°C	°C	°C/cm	(cal cm^{-2} hr^{-1})				cm^2 sec^{-1}
00	2·3	1·8	0·5	1·15	4·5	3·35	0·29	4·6
01	2·9	2·3	0·5	1·45	4·5	3·05	0·37	3·3
02	3·6	2·7	0·5	1·80	4·5	2·70	0·43	2·5
03	4·0	3·0	0·5	2·00	4·5	2·50	0·48	2·1
04	4·7	3·3	0·5	2·35	4·5	2·15	0·53	1·6
20	0·4	0·5	0·1	0·20	0·9	0·70	0·080	3·5
21	0·6	0·6	0·1	0·30	0·9	0·60	0·096	2·5
22	1·0	0·6	0·1	0·50	0·9	0·40	0·096	1·7
23	0·9	0·6	0·1	0·45	0·9	0·45	0·096	1·9
24	0·7	0·5	0·1	0·35	0·9	0·55	0·080	2·8

The values in the last column are of the same order of magnitude as those in Table 2. ike those they are averaged over a range (here 0 to 6·25 cm), but are obtained for less alm conditions and a more open crop (see Table 4). Using mean values over the periods 2 to 04, and 20 to 24, when conditions were fairly steady, temperature profiles have been rawn and, as objectively as possible, slopes measured at three values of z, with results iven in Table 4.

TABLE 4. ESTIMATED DIFFUSIVITY 18 JUNE 1956

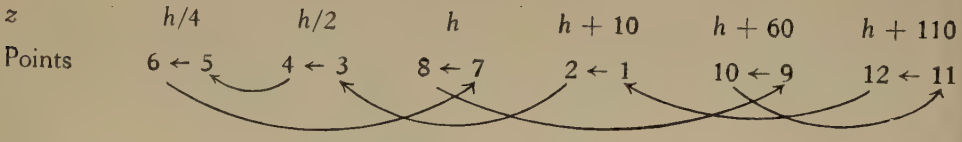
z cm	0	2	10	70	80
K cm^2 sec^{-1} (02-04)	0·6	2·5	12	19	43
K cm^2 sec^{-1} (20-24)	0·8	2·3	26	123	195

02-04, $\bar{u}_{180} = 98$ cm sec^{-1} 20-24, $\bar{u}_{180} = 212$ cm sec^{-1}

At the right-hand side of the Table are two values of K computed from the wind rofile over the crop. These cannot be very reliable because of the inversion conditions nder which the logarithmic profile has been fitted. Between 10 and 70 cm is the main af system of the crop, and it is fairly certain that over this range some conduction of ensible heat from soil to effective radiating surface at the top of the canopy takes place rough the plant and not solely through the air, so making it possible to have zero tempera- ure gradient in the air just below the top of the crop (Fig. 4 : temperature profiles at 00 nd 04 hr particularly).

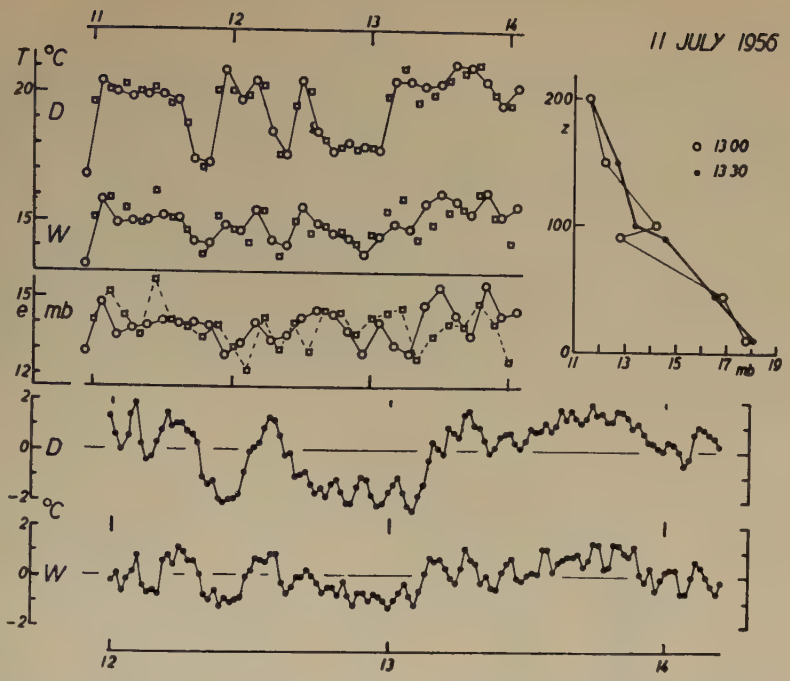
(ii) *Effect of fluctuating temperatures on interpolated profiles*

Accepting the idea that profiles of vapour pressure such as d, f, and g in Fig. 4 a anomalous, an extreme example was sought in the hope that if it could be accounted f then the same explanation might serve for less extreme cases. In the event the chose occasion turned out to be a special case, but revealed points of interest worth detailin To understand, it is necessary to know a little about the recorder. The 12 points a printed on the chart in turn with a cycle of 400 sec, i.e., about ½ min between consecutiv points. The dry-bulb elements are on the odd numbers and the wet-bulb elements o the even numbers, and while both in a pair have the same zero, every pair has a differen zero to avoid confusion on the chart. Thus between pairs, the galvanometer pointe has to swing across the chart, and the shorter this compulsory swing the more nearl the new deflexion will have been obtained when the fixed interval between printings i complete. Accordingly the arrangement, in both 1956 and 1957, was :



In an isothermal atmosphere and with a wet-bulb depression of 2·5°C at all levels the 12 points would be uniformly spaced across the chart. For the moment, the main poin to note is that the height interval to be examined (h to $h + 10$) has its thermometers recording at the maximum possible separation in time, half a period apart.

Profiles plotted for 13 hr and 1330 hr on 11 July 1956 showed very marked anomalies of opposite signs (Fig. 5, top right). On the left of Fig. 5 individual values of dry-bulb temperature (D) wet-bulb temperature (W) and vapour pressure have been plotted over a period of 3 hr from the records of traces 1 and 7, 2 and 8, with correct spacing in time for the temperature records, but treating 1 and 2 as simultaneous and 7 and 8 as simultaneous in deriving vapour pressures. Normal expectation is that the circles (for $z = h$) should lie above the squares (for $z = h + 10$), in the vapour-pressure trace, yet it is clear that over relatively long periods the dashed line joining the squares lies above the continuous line joining the circles. Examination of the temperature record (D) shows that the continuous line is not a bad fit for the squares too, and that with very little distortion it would be possible to join up all the points into one sequence, i.e., that the temperature is much the same at the two levels, and apparent differences have arisen from differences in the phases of fluctuations at the imposed sampling times. To test this possibility the assumption was made that the fluctuations at all six levels were in phase within a few seconds; and for each level the mean temperatures (dry-bulb and wet-bulb) were determined over a period of 20 cycles (9 cycles = 1 hr). The deviations from the means were then plotted in time sequence, on twice the previous scale, dry-bulb temperatures in one group (D), and wet-bulb temperatures in another (W). A similar plot of deviation divided by mean deviation gave the same sort of picture, and is not reproduced. Though part of the effect may be a spurious result of the way the data have been handled, it does seem clear that over this period of 2 hr there were occasions when air temperature fluctuated with a period very close to that of the instrument, the wet-bulb showing a less well-marked periodicity not in phase with the air temperature. The periodicity itself could put apparent distortions into otherwise regular temperature profiles, and as the amplitude is of the order of a few tenths of a degree Celsius the scale is about right if it is to account for the type of temperature profile shown in Fig. 4. There will be corresponding distortions of wet-bulb temperature profiles, and, if, as seems to be the case, the fluctuations responsible are not always in phase with dry-bulb temperature fluctuations, then the distortions in vapour-pressure profiles could be greater than those in the temperature profiles.



gure 5. Top left : dry-bulb (D), wet-bulb (W) temperatures, and vapour pressure (e) at 90 cm (circles) and 100 cm (squares) over 3-hr period. Crop height 90-cm.

Top right : vapour profiles at 1300 and 1330 derived from interpolated readings.

ttom (on twice-time scale) : departures from 2-hr means of dry-bulb (D) and wet-bulb (W) temperatures at all six levels of recording (four above, two below top of crop).

As a contribution to study of fluctuations this discussion is completely inadequate, ıt its immediate value is clear : during periods of fluctuating temperature the equipment nnot provide reliable instantaneous profiles, and any discussion of profile physics must e based on relatively long-period averages. In later sections the period used is 4 hr.

5. Results for 1957

1. Variations across the site

In addition to the ' standard profile ' based on two masts 25 cm apart (Fig. 1, (d) ıd (e)), there were wet- and dry-bulb temperatures measured at constant height at six ɔsitions equally spaced, with cup anemometers at four of them, set at 100 cm above ound level. The anemometers were left untouched, but the thermometer units were oved from time to time, and sets of records obtained during a period of a few weeks. addition, further useful evidence was obtained from what Long (1957) referred to as e dew-point (T_d) recorder, and the dew-point gradient (ΔT_d) recorder. The first corded on the chart the value of $2T_w - T_a$, where T_w is the wet-bulb temperature, is the dry-bulb temperature, with $2T_w - T_a \simeq T_d$. This is a crude approximation, in neral, but the difference between two such values at different values of z is a little more act and *is rarely wrong in sign*. For two levels, z_1 and z_2, the ΔT_d recorder has given lues of

$$\Delta T_d = (2T_{w1} + T_{a2}) - (2T_{w2} + T_{a1}),$$

d has been useful, particularly at night, in giving the sign of the vapour-pressure gradient tween crop canopy and air above, or crop canopy and air below. Long (1957, Fig. 4) ves an example of its use in showing a short period of dew formation in the middle of e night, and (1958) has discussed the frequency with which such dew has completely -evaporated before the official hour of observation at crop weather stations.

(i) *Wind*

Records were taken at sites g_3, g_4, g_5 and g_6 (Fig. 1), and Table 5 shows selected values for periods of 1 hr on 18 and 19 June 1957, when the anemometers were at 100 cm above ground and the crop heights were 71 cm (09 hr 18 June) and 74 cm (09 hr 19 June). Although the middle of the crop was fairly uniform in height this was not a bowling green and the different responses of the anemometers is probably a mixed reaction to differences in fetch over the crop; differences in height of crop, itself a function of wind speed and probably of wind direction too; and differences in anemometer performance. The mean hourly wind speed has been expressed as a ratio of that at the most northerly of the points and in choosing illustrative examples extremes have been sought, e.g., the calmest and the windiest hours, the hours showing most variation and least variation in ratio, and hours showing an indication of a gradient. The means at the foot of Table 5 are for all periods (24 hr 18 June, 22 hr 19 June).

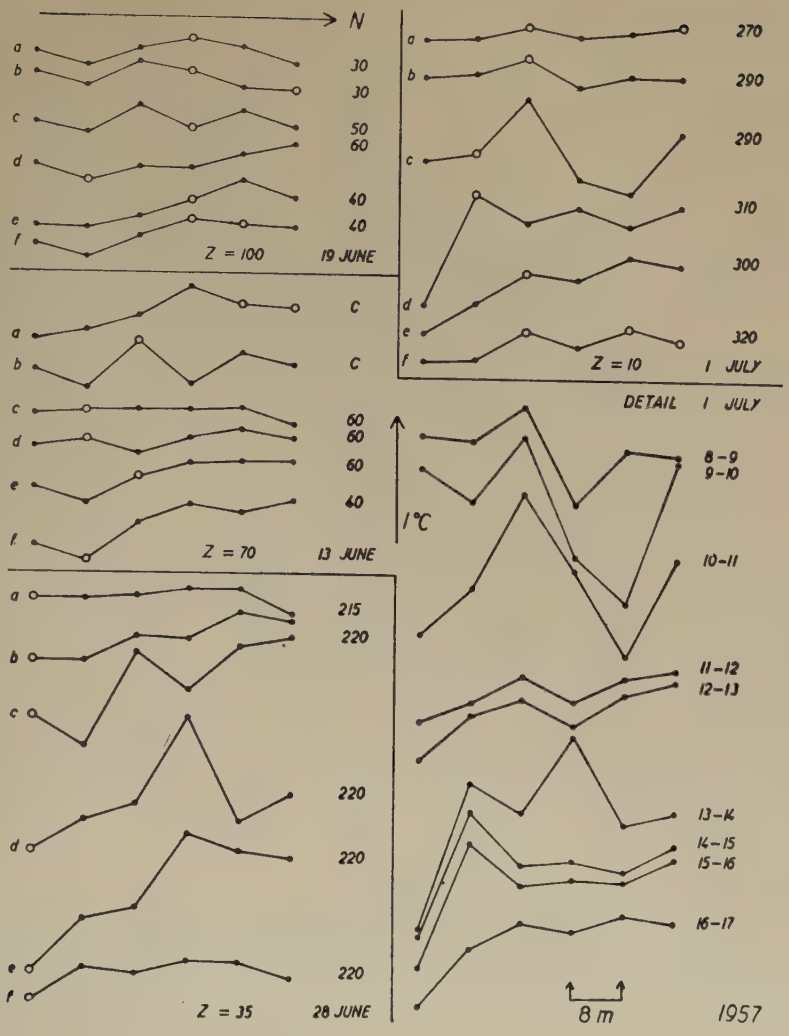
TABLE 5. VARIATION OF WIND SPEED ACROSS SITE

Day	Period	Position				Direction ($N = 360$) at 10 m	Character
		g_3 u_3/u_6	g_4 u_4/u_6	g_5 u_5/u_6	g_6 u_6 cm sec^{-1}		
18 June	00-01	1·08	1·08	1·06	100	20	calmest
	03-04	1·13	1·10	1·05	116	20	extreme range
	07-08	0·99	0·94	0·95	150	15	nearest to N-
	11-12	0·94	0·95	0·98	246	40	gradient ?
	15-16	0·97	1·02	1·00	274	30	maximum
	19-20	0·97	1·00	1·00	216	30	most uniform
Mean	00-24	1·019	1·035	0·996	186		
19 June	01-02	1·08	1·09	1·05	130	30	calmest
	07-08	0.86	1·00	0·95	240	35	extreme range
	11-12	0·99	1·02	0·99	299	50	maximum
	16-17	0·99	1·00	0·98	211	70	most uniform
	20-21	0-95	0·96	0·96	227	40	gradient ?
Mean	00-24	0·991	1·013	0·981	215		

Associations between performance and wind direction are not very strong. It *may* be that under the calmest conditions (00-01, 03-04 hr 18 June, 01-02 hr 19 June) there is an increase in velocity along the north-south line of the anemometers; it *may* be that one of the nearly uniform responses (16-17 hr 19 June) is causally linked with a wind direction most nearly at right angles to the line of anemometers; but it is probably safer to take a pessimistic view and regard the variance as a measure of instrumental uncertainty. Though possible errors of the order of 1 per cent have little effect when trying to draw profiles, they introduce major difficulties in fitting profiles with equations : it becomes all too easy to make every profile a logarithmic profile.

(ii) *Air temperature*

Fig. 6 shows mean values over 4-hr periods (each point the mean of 36 readings) on four days when the psychrometer array was at varied heights; on the first two days it was above the top of the crop by 26 and 10 cm respectively, and on the second two days it was below the top of the crop by 55 and 85 cm respectively. All were sunny days though the last (1 July) was overcast for 2 hr at midday. The vertical scale of temperature difference here is five times that on previous figures. In comparing the curves it may be important to know that there was heavy rain on 24-25 June (0·7 in.) and on 30 June (0·6 in.)



gure 6. Variations in temperature at constant level across the site. Interval 8 m. (a) Mean 00-04, (b) 04-08, 08-12, (d) 12-16, (e) 16-20, (f) 20-24. Crop heights : 19 June, 74 cm; 13 June, 60 cm; 28 June, 90 cm; July, 95 cm. Open circles represent positions of maximum vapour pressure in the same period (see Table 6). Wind direction on the right ($N = 360$).

that the curves for 1 July, close to the ground, may be somewhat buffered by the wet round just below the elements. The points are plotted from south (left) to north (right), nd approximate wind direction is indicated on the right of each curve (C = calm). Iaximum and minimum screen temperatures in the nearby meteorological enclosure were, the order of the figure, 19 June, 77 and 52°F, with 10·2 hr sunshine; 13 June, 73 and °F, with 15·3 hr sunshine; 28 June, 85 and 52°F, with 15·6 hr sunshine; 1 July, 75 and °F, with 9·1 hr sunshine.

The field calibration on 12 June, with all thermometers in stirred water (temperature nge during test 10-22°C), showed no differences greater than 0·1°C; a check on night inima (see Table 6) showed only a slightly greater scatter; and a further comparison uring the nearly uniform temperature over the rain period from 18 hr 24 June to 06 hr June, when everything was wet, showed no important differences between the thermo-eter readings. The much greater differences that characterize most of the sets of data n Fig. 6 are therefore regarded as real differences between the temperatures of the sensing

elements. Though it is impossible to assert that all 12 elements had precisely the same exposure, and hence might be expected to have the same radiation error (if any), it seems to be a reasonable working assumption, i.e., there is a high probability that the differences in Fig. 6 are differences in air temperature. If the left-hand values are disregarded – anomalies are expected at the edge of the plot anyway – then the maximum difference over a period of 4 hr is 0·8°C at two points 8 m apart, and at about half the crop height above ground (28 June). The position and persistence of warm spots probably depend upon accidents of placing of the elements with respect to openings in the leaf canopy and the persistence of direct sunshine falling on the ground near the element or on a part of the plant near the element. As the sun moves round and its elevation changes so the warm spots will move. One of the most striking changes is between the curves for 8-12 (period c) and 12-16 (period d) on 1 July (top right Fig. 6), and is seen more clearly in the detailed curves, below, for hourly periods from 8-9 to 16-17. The size of the fluctuations is, of course, much greater : the biggest range is 1·3°C averaged over the hour 10-11, and the shift in the warm spot between the sunny periods before and after the mid-day cloudiness is well marked. The horizontal scale on which the differences might arise could be quite small, so that the trends of Fig. 6 might be reproduced by elements at about a metre apart. Horizontal mixing probably reduces this risk somewhat, and the gradual increase in the scale of the fluctuations as z decreases is probably an indication of a reduction in the intensity of mixing from above the crop downwards. Remembering that these four sets of records are for different days the comparison can only be semi-quantitative, but it does seem clear that the fluctuations are least where the mixing is greatest. It was hoped that this kind of observation might show some evidence of advective effect, but apart from a slight suggestion in some of the curves for 19 June and 13 June which show a trend of decreasing air temperature downwind, there is no really conclusive evidence of the air being cooled during its daytime movement over the experimental site. Further comment will be made when the vapour-pressure measurements at constant height have been reviewed.

(*iii*) *Vapour pressure*

All the temperature values plotted on Fig. 6 are means of 36 (or 9 for bottom right) readings. In computing vapour pressures for this section the 4-hr mean values of dry-bulb and wet-bulb temperatures have been used except for the first two entries in Table 6, which are for shorter periods. For these, the night minima, over a period of $\frac{1}{2}$ hr, of both temperatures were averaged for groups of three nights when site variations might be expected to be at their lowest.

Only the highest and lowest levels of recording are represented in Table 6, and the entries reveal that the fluctuations in vapour pressure are of the same order as those of air temperature, taking a change of 1 mb as equivalent to a change of 1°C, with the smallest fluctuations again at the level of maximum mixing. The pattern, however, is not quite the same, for on Fig. 6 the sites of maximum humidity have been distinguished by large open circles and it will be seen that they are not always the sites of maximum temperature, as they would be were the temperature differences solely caused by radiation errors. The range of fluctuations in the middle of the crop is about 5 per cent of the mean vapour pressure, and for the last set of data (1 July) corresponds to about 5 per cent range in relative humidity. There is just a suggestion that this parameter is more conservative across the site than is absolute humidity, and shows a more uniform change than either temperature or vapour pressure, but in the single example given, at 10 cm above wet soil, this may be an indication of buffering action by the soil. Again, as for the temperature record, there is no clear evidence of increasing humidity downwind, and accepting the temperatures of Fig. 6 and the vapour pressures of Table 6 as true values it seems that advective effects that might be expected by working close to the upwind edge of a crop are too small to be detected in the presence of local fluctuations in and above the crop.

TABLE 6. VARIATION OF HUMIDITY ACROSS SITE

Position	g_1	g_2	g_3	g_4	g_5	g_6	Range of $g_2 - g_6$
nima 16, 17, 18 June 1957 $z = 70$ cm $h = 68$ cm							
ean T_a °C	11·27	11·13	11·31	11·33	11·33	11·25	0·20°C
ean e mb	13·1	13·1	13·1	13·1	13·1	13·15	0·05 mb
nima 19, 20, 21 June 1957 $z = 100$ cm $h = 76$ cm							
ean T_a °C	10·13	9·99	10·16	10·21	10·15	10·00	0·22°C
ean e mb	11·45	11·45	11·4	11·45	11·5	11·5	0·10 mb
erages e mb 19 June 1957 $z = 100$ cm $h = 74$ cm							
0-4	11·95	11·9	11·85	12·0	11·95	12·0	0·15
4-8	12·85	12·8	12·75	12·85	12·8	12·85	0·10
8-12	11·4	11·5	11·35	11·45	11·3	11·3	0·20
12-16	8·1	8·3	8·1	8·3	8·1	7·9	0·40
16-20	8·5	8·7	8·7	8·95	8·6	8·6	0·35
20-24	9·85	9·8	9·8	9·9	9·9	9·8	0·10
erages e mb 1 July 1957 $z = 10$ cm $h = 95$ cm							
0-4	17·9	17·9	18·05	17·9	18·0	18·05	0·15
4-8	19·25	19·25	19·4	19·1	19·2	19·2	0·30
8-12	22·0	22·8	22·6	21·8	22·1	21·9	1·00
12-16	21·8	22·7	21·9	22·1	21·8	21·6	1·10
16-20	19·9	20·15	20·3	19·8	19·85	19·7	0·60
20-24	15·75	15·55	15·8	15·75	15·8	15·8	0·25
erages relative humidity (per cent) 1 July 1957							
0-4	100	100	100	100	100	100	0
4-8	100	100	100	100	100	100	0
8-12	88·5	91	88	88·5	90	86·5	5
12-16	80	79	77	77·5	77	76	3
16-20	87	87	86	84·5	83·5	83	4
20-24	99·8	98·5	98·5	99	98·5	99	1

(*iv*) *Part profiles*

Much more evidence of these horizontal variations is available in the many *ad hoc* servations that have been made. Above the crop two 'dew-point recorders' have en run at the same heights as standard-profile instruments, and the direct record of $T_w - T_a$ compared with the value calculated from individual values of T_w and T_a the same height. Discrepancies of a few tenths of a degree Celsius exist, and persist r hours at a time with the same sign.

The 'dew-point gradient recorders' have *not* been run at the standard heights, it were arranged with one set of elements in the thickest part of the canopy and another t either well above the canopy or at 10 cm above the ground. Comparison with standard ofiles involved plotting $2\,T_w - T_a$ for the standard profile, and then seeing if the direct easures of $\Delta\,(2\,T_w - T_a)$ could be made to fit. Except at night, they rarely could, th the bigger discrepancies between canopy and air above, as might be inferred from g. 6, both in magnitude and persistence.

(*v*) *Conclusion*

Although the information in the last few sections has its own intrinsic interest, the ctions grew out of an attempt to explain, and if possible remove, the anomalies of Fig. 4. his attempt has been only partly successful: the exaggeration of the profile contrast top right in Fig. 5 can be removed by recognizing that the resolving power of the corder is not adequate for study of short periods, but even a period of adequate length

in this connexion may be still too short to take care of long-persisting differences in th values of the weather parameters at constant height across the site. The concept of ' profile ' as a guide to analysis of mixing and transport is based on the assumption tha what happens at a height z is dependent upon surface conditions upwind at a distanc x, say, and that for a fully-developed profile it does not matter where the point $x = 0$ placed, provided that the value of the parameter at $z = 0$ is the same for all values of x For a short crop, or with none, the assumption is unchallenged, but it seems clear that fc a tall crop, such as the wheat in these experiments, it is unrealistic, and that some distortio in vertical profiles must be accepted as an inevitable consequence of non-uniformit in the horizontal at crop level. This diagnosis is not new, for Suomi and Tanner (1958 found it worth while to use a rotating boom to carry their instruments round in horizonta circles, so getting short-period spatial averages of parameters at constant height, and elimi nating the effects of the patchiness of the grass cover on their site.

Part II. *Analytical and speculative*

6. On profiles

The vertical transport of momentum, heat and water vapour can be computed fro measurements of the relevant potentials and their gradients, the most confident estimate being for an adiabatic or neutral atmosphere in which experience shows the wind spee over a non-flexible surface to be given by

$$u = \frac{u_*}{k} \ln \frac{z - d}{z_0}, \qquad (1$$

where all symbols have their usual meanings. The computation assumes a similar logari thmic variation with height of the temperature and vapour pressure, and this identit of profile shape over a short crop cover appears to be well established by observations c Pasquill (1949), Rider and Robinson (1951), Halstead (1954), Rider (1954a) and Elnes (1956). There have been criticisms of the identity, such as that by Priestley (1952) and of the application, for example in Holland where Dutch workers on the Rottegats polder are not yet satisfied with the agreement between theoretical estimates of evaporatio and measured values for the polder, though there are uncertainties in the measurement too (Deij *et al.* 1955).

For a tall crop such as the wheat in the present series of observations it is expecte that the value of d, the height of the virtual sink for momentum (more correctly at $d + z_0$ will be of the order of the height of the crop, but *a priori* it seems somewhat unreasonabl to expect the same value of d to represent the height of the virtual source of water vapou when the real source is extended in depth and probably not of equal strength at all levels or to represent the height of the virtual source (or sink) for either sensible or radiant hea energy. (Bjorgum 1953, Halstead 1954, Sheppard 1958).

The approach to the profile problem was, therefore, to see if different values of might be needed for momentum, vapour and heat transfer, using the integrated form c the Deacon equation, discussed by Sutton (1953) and Sheppard (1958), which, for win velocity, can be written

$$u = \frac{u_*}{k\alpha} \left[\left(\frac{z - d}{z_0} \right)^{\alpha} - 1 \right], \qquad (2$$

where $\alpha = 1 - \beta$ of Deacon (1949). On expansion this gives

$$u = \frac{u_*}{k} \left[\ln \frac{z - d}{z_0} + \frac{\alpha}{2!} \ln^2 \left(\frac{z - d}{z_0} \right) + \ldots . \right] \qquad (3$$

leading to Eq. (1) when $\alpha = 0$. It was hoped that for each profile values of α and d migh be obtained for each of the three potentials, and identity or non-identity thereby established

eneralizing the potential as P_1, P_2, P_3, P_4 measured at four heights z_1, z_2, z_3, z_4 the iantity

$$F(4, 3, 2) = \frac{(z_4 - d)^\alpha - (z_3 - d)^\alpha}{(z_3 - d)^\alpha - (z_2 - d)^\alpha}$$

$$= \frac{P_4 - P_3}{P_3 - P_2},$$

d the corresponding $F(3, 2, 1)$, were computed for a range of values of α, d and z. It needed only a cursory inspection of the temperature and vapour-pressure profiles, ch as those in Fig. 4, to dispel any ideas of getting α and d from them, but there remained pe of success with the wind profiles, though here too there was a discouraging warning m the experiences of Rider (1954b) on oats and of Inoue (1955) on rice, who found that aving of the crop was a disturbing factor. Mean values of u_4, u_3, u_2 and u_1 over periods an hour were used to compute $F(4, 3, 2)$ and $F(3, 2, 1)$, and the values of α and d ught that simultaneously satisfied the equations. After working through four days' cords without any success, the attempt to use hourly values was abandoned. There as no worthwhile improvement when mean values over periods of four hours were ed, this being attempted for the eight days 12-19 June 1957 which provide the material most of the present section. It was only when a week's data were put together, kept in hr periods, i.e., using means over 28 hr, that success was achieved. (See discussion of g. 8 later in this paper).

The direct attempt at establishing identity of profiles having failed, an indirect ethod was tried, with success. The application is considered in detail later : here, ly the principle is given, using wind velocity and vapour pressure as one of the three ssible examples. If $u = a f_1(z)$ and $e_0 - e = b f_2(z)$, then if u and e are plotted one ainst the other over a range of values of z, the points will lie on a straight line if $f_1 \equiv f_2$, hatever the form of f. Given such a straight line, and evidence that $f = \ln(z - d)/z_0$, en it is possible to use the straight line to compute the vapour-transfer rate (or the nsible-heat transfer rate, using u and T), effectively in the way used by Rider (1957).

7. Wind profiles 12-19 June 1957

This period of eight days provides the data for the detailed analysis of the next three ctions. It was characterized by clear skies, high temperatures, and, for part of the eriod, by a very dry atmosphere. One object in choosing it was not achieved : there ad been rain on 7, 8, 9 and 10 June, and it was hoped that the effect of the steady transi- on from wet-surface soil to dry soil might be picked up in the vapour pressure and tem- erature profiles.

Fig. 7 gives the wind profiles for the eight days, split up into 4-hr periods (00-04, 4-08 and so on), the vertical scale of height being linear. The top four points on each rofile are obtained from the recording cup anemometers, and it will be noted that the eights changed daily as the anemometers were moved every morning to keep pace with a op growing 2 to 3 cm taller every day (see Fig. 1, bottom right). The single observation the crop is from the hot-bulb anemometer, the level of which was changed from $z = 25$ $z = 35$ cm on 17 June. It is assumed that $u \equiv 0$ at ground level. At the bottom of ery other curve is an indication of wind direction during the 4-hr period, taken from the ines anemometer 75 m away. The direction of the rows was 90-270. Except for the latively calm day of 12 June, the curves in bulk show the characteristic diurnal cycle wind speed. The curves for periods (a) and (b) on 15 June are probably misleading : is almost certain that the wind speeds, as plotted, just above the crop (open circles) e too low because the crop had grown sufficiently during the night for the tips of the aves to interfere with the rotation of the anemometer cups.

During the very calm weather at the beginning of the week, particularly in periods) and (b) of 13 June, the airflow appeared to be very nearly laminar from the ground up.

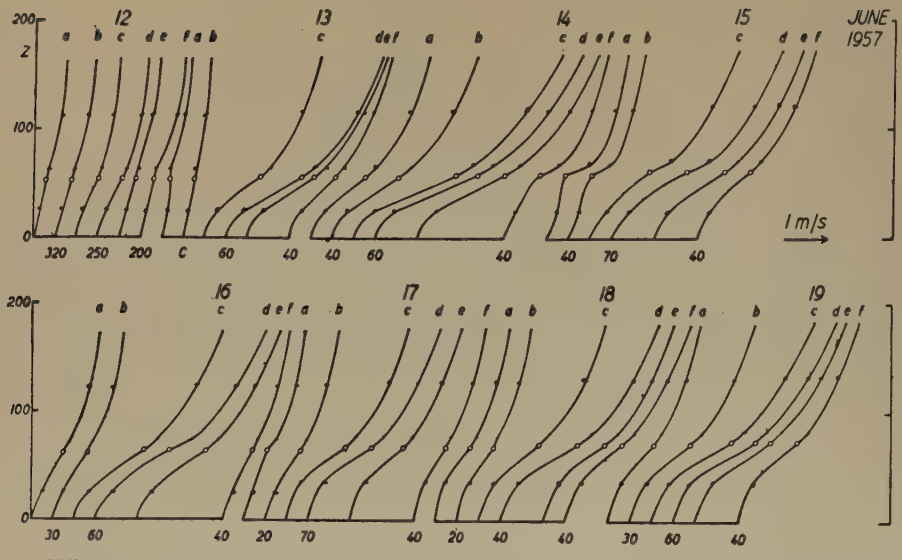


Figure 7. Wind profiles, all starting from zero at ground level. (a) Mean 00-04, (b) 04-08, (c) 08-12 (d) 12-16, (e) 16-20, (f) 20-24. The open circle represents the observation just above the top of the crop

Ramdas (1946) working with much denser and taller crops such as sugar cane, noted that in similar calm the motion of air inside the crop, though not completely absent, was very much suppressed with negligible turbulence, the random movements being very sluggish with a horizontal drift in evidence; and that inside the crops there was a decrease of wind velocity with height because the wind-break effect was greatest at the level of maximum density of foliage. The probable shape of the profile in calm and in moderate wind is considered later (Fig. 9), but first it is convenient to apply standard analysis to the profiles over the crop.

(*i*) *Wind over the crop*

Except for the first day of the period there is a fairly consistent daily pattern in phase and amplitude of the profiles, and Fig. 8 has been prepared from the 7-day (13-19) means of 4 hr periods. The ordinate is wind velocity, u, (mean of 28 hr) and the abscissa is ku/u_*, obtained as follows : for one set of points the logarithmic profile (Eq. (1)) has been fitted, by estimating values of d for $\alpha = 0$ from the prepared curves of $F(4, 3, 2)$ and $F(3, 2, 1)$, taking the mean, plotting $\ln(z - d)$ against u, reading the intercept $\ln z_0$ at $u = 0$, and then determining $f(z) = \ln(z - d)/z_0$; which is the abscissa. For the second sets of points, values of α were chosen according to Richardson number from the Deacon curve reproduced by Sutton (1953 p. 237; bottom right, Fig. 8), and in calculating R_i the following short cuts were taken :

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial \ln(z - d)} \cdot \frac{1}{(z - d)},$$

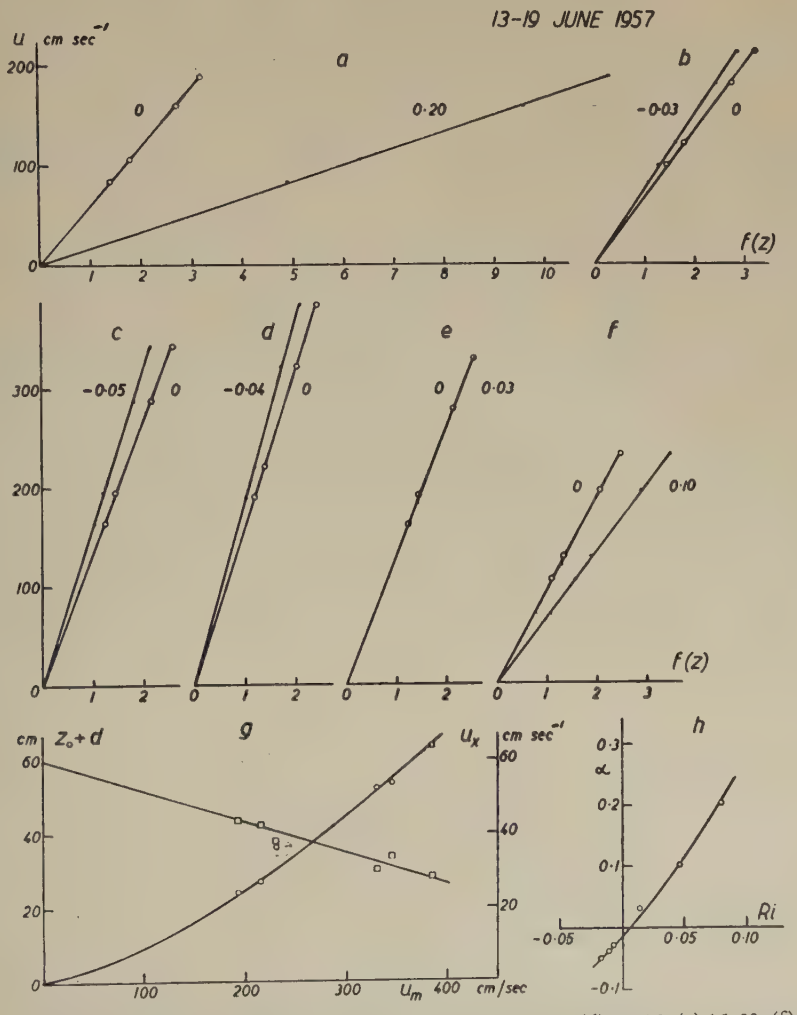
where the gradient needed on the right-hand side is the slope of the first set of points just described;

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial u} \cdot \frac{\partial u}{\partial z},$$

where $\partial T/\partial u$ is obtained from a later diagram (Fig. 10). With a value of α chosen, the value of d was taken from $F(4, 3, 2)$, $(z - d)^\alpha$ plotted against u, the best straight line drawn, leading to $z_0{}^\alpha$ (at $u = 0$) and thence $f(z) = \frac{1}{\alpha}\left[\left(\frac{z - d}{z_0}\right)^\alpha - 1\right]$ determined; which is the abscissa.

If choice of equation is to be determined by goodness of fit to the expected straight ne, there is nothing to choose between the two sets of points. The deviation in the ertical is of the same order as the variations noted in the 24-hr means of u in Table 5. he deviation in the horizontal corresponds to an error of about 1 cm in the spacing of the vo lowest anemometers, nominally at $\Delta z = 10$ cm apart. It seems that this spacing, nosen primarily for study of the temperature and humidity differences, may be too small r anemometers having cup diameters of 5·5 cm.

The slope of the line is u_*/k, and for most of the profiles the differences between ne two are probably unimportant, but for the period of strong inversion the ratio of the opes is nearly 4 : 1, and a twelve-fold ratio in estimated shearing stress cannot be ignored. oth estimates cannot be true, but both can be false; that based on the logarithmic profile ecause the statistically good fit here may be physically unrealistic, and that based on Eq. (2) ecause the quantity u_*/k on the right-hand side was put there to give the right answer at = 0. Other quantities are more nearly equal, and some are given in Table 7, with raphical representation of inter-relations for $\alpha = 0$ in the left-hand diagram at the foot f Fig. 8.



igure 8. Mean wind profiles for 7 days. (a) 00-04, (b) 04-08, (c) 08-12, (d) 12-16, (e) 16-20, (f) 20-24 – each eing mean of 7 × 4 hr. Abscissa : $f(z) = \frac{1}{\alpha}\left[\left(\frac{z-d}{z_0}\right)^{\alpha} - 1\right]$. Curve 0 is for $\alpha = 0$ (logarithmic profile) : he other is for α taken from diagram at bottom right. Bottom left : roughness and shearing parameters as a function of wind velocity at highest level of recording.

TABLE 7. DAILY CYCLE OF WIND PARAMETERS 13-19 JUNE
(Mean crop height ≃ 60 cm)

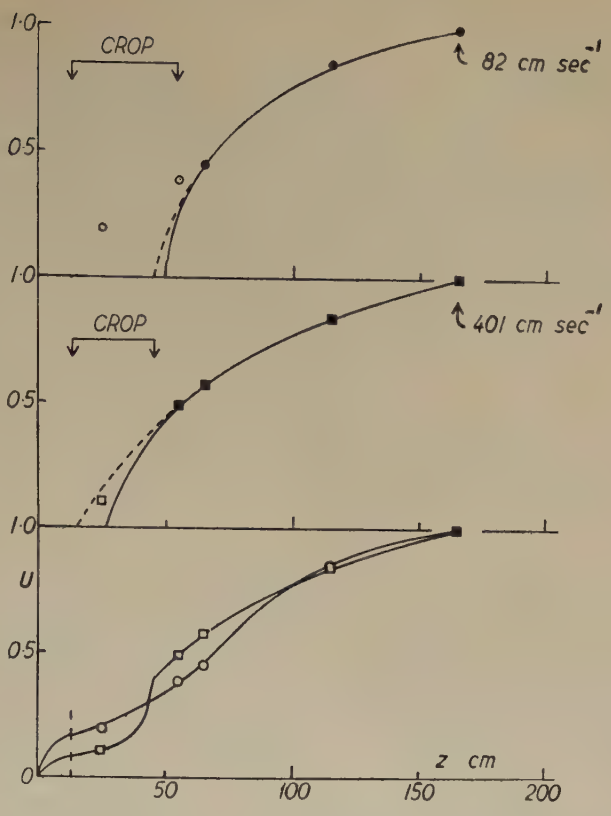
	Period	Ri	α	d cm	z_0 cm	z_0+d cm	u_*/k cm sec^{-1}	u_m cm sec^{-1}	α	d cm	z_0 cm	z_0+d cm	u_*/k cm sec^{-1}
a	0-4	0·080	0·0	38	5·4	43	59	190	0·20	50	0·3	50	17
b	4-8	− 0·007	0·0	37	5·4	42	67	216	− 0·03	35	6·6	42	75
c	8-12	− 0·017	0·0	22	11·0	33	132	342	− 0·05	14	15·8	30	158
d	12-16	− 0·010	0·0	14	13·7	28	158	384	− 0·04	8	17·8	26	181
e	16-20	0·014	0·0	18	12·2	30	130	331	0·03	18	12·3	30	127
f	20-24	0·046	0·0	25	11·8	37	92	232	0·10	30	7·0	37	66

The values of $z_0 + d$ are nearly equal, as they ought to be from the way the data have been handled, and the level of this virtual sink for momentum is about half-way down into the crop at the time of maximum wind speed, and nearer the top at minimum wind speed. Extrapolating to zero wind speed (Fig. 8) gives a value near $z_0 + d = 60$ cm – the top of the crop. The components move in anti-phase, the roughness being in phase with wind speed, and the zero-plane displacement out of phase with wind speed.

These data for wheat are in conflict with the first results of Inoue (1955) for rice though the same tendency ought to be manifest. He then reported, over rice under nearly adiabatic conditions, that d had a maximum (and z_0 a minimum) value at a critical wind velocity near 550 cm sec^{-1} at 2 m height consequent on the coincidence of a turbulence frequency (not clearly specified in his English summary) and the natural frequency of vibrating rice (*c.* 1 per sec). Table 7 suggests just the opposite, and indeed Tani, Inoue and Imai (1956) in a later paper have contradicted the earlier statement, now showing a decrease of d (and increase of z_0) with increasing wind velocity on about the same relative scale as that in Table 7. ' The cause of the discrepancy seems to be due to the different condition of rice stalks in growth stage, density, flexibility etc.' The possibility of some sort of resonance being detected from wind profiles is of considerable interest, and the phenomenon may be of practical importance later in the life of the wheat plant when in full ear. Wetted by rain, the natural period will be longer, and presumably the plant will then respond more violently to larger eddies, and conceivably the amplitude of vibration might become so big in relation to the rigidity of the stalk (perhaps reduced already by fungus disease) that displacement beyond the angle of no return occurs and the plant will ' lodge.'

(*ii*) *Wind in the crop*

Ultimate biological interest will be more concerned with ventilation *in* the crop as as factor in the transfer processes between leaves and their environment, particularly the uptake of carbon dioxide at various levels. Pending a fuller analysis, which will need more detailed measurements, it is possible to give a qualitative picture now, adequate as a basis for a later discussion (§ 11). From the profiles of Fig. 7, two groups of six have been selected in the period 12-14 June as the six calmest and the six windiest, and mean profiles have been computed, normalized to make the average velocity at the highest level of recording ($z = h + 110$) unity. Fig. 9 shows these mean profiles with $U = u(z)/u(165)$, first for calm and windy separately, and then with both together. In the top diagram only the three full points have been used to fit a logarithmic profile, drawn as the full line reaching $u = 0$ at $z = z_0 + d = 49{\cdot}5$ cm. If the crop were behaving as a raised platform of the same roughness, its level would be at $z = d = 45$ cm, and the dashed line indicates the probable ' real ' profile over such a platform.



gure 9. Normalized wind profile in calm, and moderate wind. Top pair : fitted logarithmic profiles. ttom : probable shape down to ground. Crop height in calm, 55 cm; in wind, assumed 45 cm.

In the second diagram all four of the above-crop wind observations are easily fitted ith a logarithmic profile, and the two extrapolations to $u = 0$ are to $z = z_0 + d = 26{\cdot}5$ cm ull), and $z = d = 15$ cm (dashed). On both of these diagrams is a rough indication of e thickness of the crop, the lower limit being the same for both, as – approximately – the st quarter of the height of the plant is clear stalk with no leaves. The upper limits ve been made different, the ' calm ' height being the stated height, the ' windy ' height ing drawn 10 cm lower, an arbitrary choice to indicate that when the wind blows the ants bend, and tend to produce a continuous horizontal surface. As a guide to orders of agnitude, the volume of the crop is about 1/500 of the volume of air it occupies : the rface (counting both sides of the leaves) is about four times the area of the ground on hich the crop stands, distributed in an array of parallel palisades 13 cm apart (Fig. 1).

In calm conditions there is little doubt that the air can move easily between these lisades, with some decrease of shearing stress at what might well be a fairly uniform te between the heights $h > z > h/4$. Below $h/4$ there is probably little further decrease shearing stress until the surface is reached. In windy conditions there is no decrease in earing stress until the level $h - 10$ is reached, and then it must be exceedingly rapid, ., there must be a very steep velocity gradient in the upper part of the crop, settling down something like the same value as that under calm conditions in the lower part. It is ese general considerations that have led to the profiles drawn in the bottom diagram. nough the shapes are suggestive only, there is no doubt that they are different, and, as awn, they do appear to satisfy a conservation principle, namely that the areas under e curves should be approximately equal (ignoring minor differences above the highest vel of recording).

8. Temperature and humidity profiles 12-19 June 1957

Anticipating a result yet to be established, the temperature profile is, statistically, the same shape as the wind profile; and there is a similar identity between vapour pressure and wind profiles. As the wind profile, at least in the periodic means, can be represented by Eq. (1), the following simple analysis will help to explain the meaning of Fig. 10 and Fig. 11, and the quantitative estimates of fluxes made from the diagrams.

From Eq. (1) the plot of u against $\ln(z - d)$ gives $\ln(z_m - d)/z_0$ direct, where z_m is the height of the highest observation level. Plotting u against e (or T) gives a straight line whose intercept on the e (or T) axis is e_0 (or T_0) say. If the value at the highest level is e_m (or T_m), then the evaporation rate (E) and the sensible heat flux (Q_s) are :

$$E = C_1 \frac{u_m (e_0 - e_m)}{[\ln (z_m - d)/z_0]^2}, \quad . \quad . \quad . \quad (4)$$

$$Q_s = C_2 \frac{u_m (T_0 - T_m)}{[\ln (z_m - d)/z_0]^2}, \quad . \quad . \quad . \quad (5)$$

where C_1 and C_2 are constants depending on the units employed.

(i) *Temperature*

Fig. 10 gives eight days' records, and general means, of wind speed as ordinate and air temperature as abscissa averaged over 4-hr periods (36 observations per point for temperature) with each plot displaced to the right of its predecessors to avoid overlap. Though this displacement is not uniform it does result in the envelope of the tops of the lines presenting a coarse picture of diurnal changes in wind speed, and shows the first day as much the calmest of the eight. The four upper points are for levels at or above crop height, and the fifth is for a level in the crop at which both parameters were measured. The sixth is based on measured temperature and an estimated value of u taken from Fig. 7 at the appropriate level ($z \simeq 10$ cm).

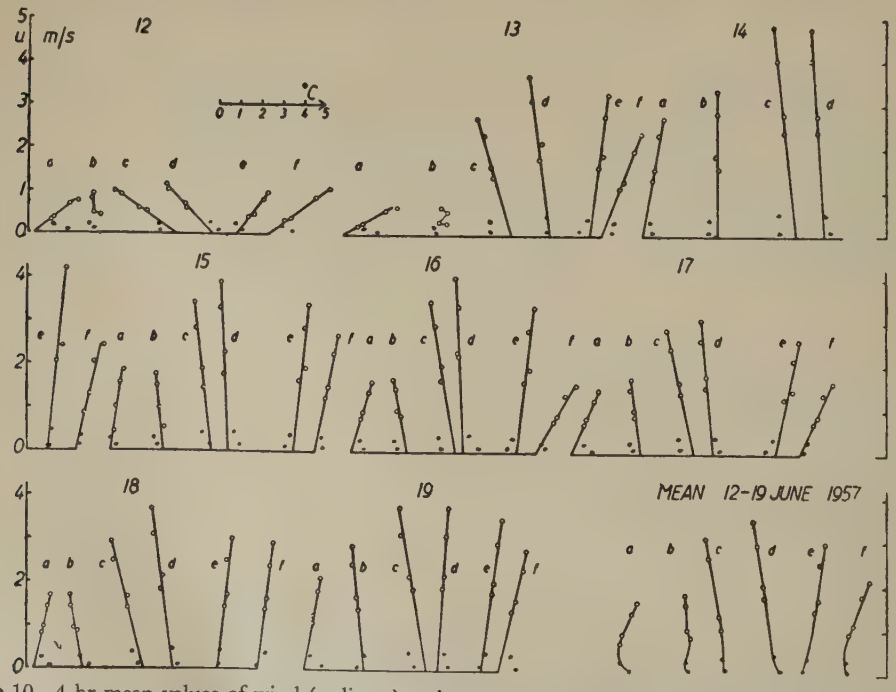


Figure 10. 4-hr mean values of wind (ordinate) and temperature (abscissa) (a) 00-04, (b) 04-08, (c) 08-12, (d) 12-16, (e) 16-20, (f) 20-24. Bottom right : eight-day mean. Open circles : above crop, used for drawing straight lines. Full points : inside crop, ignored except in general mean.

With minor differences the daily pattern is clearly shown in the mean data in the ttom right-hand corner of Fig. 10, namely a marked inversion 00-04, period a; a transi-n period after sunrise, giving, on average, a nearly isothermal atmosphere 04-08, riod b; a lapse in each of the periods (c and d) 08-12, 12-16 with different gradients the crop just above the soil surface; and the return of the inversion (e and f) 16-20, -24. On most days, and very easily on the first, it is possible to detect the effect of the ing sun warming the top of the crop, but at 06 hr the sun was shining along the rows d from then on, as the sun's altitude increased, it seems clear that the soil surface has en the main absorber and emitter of radiation in the heat exchanges, in marked contrast ith the behaviour of the dense crop represented in Fig. 2. Conditions during the official calm " of 04-08 on 13 June appear to have been those of a complete overturn of the layer air above the crop, lasting about 1 hr, on the evidence of an analysis of plots of wet-ılb temperatures against dry-bulb temperatures.

Quantitatively the most important points are the top four; and again the main discus-on can be based on the 8-day means. In the light of the known variability of temperature constant level, demonstrated in Part I, each of the six curves (either for the average day for real days) must be fitted as a straight line above crop level, at least for the four to d covering the period 00-16. For e, 16-20, clearly, and for f, 20-24, possibly, the ıttern is slightly different in showing the air a little warmer at $h + 10$ cm and a little ooler at $h + 60$ cm than the straight line relationship would suggest. Thorough examina-on of the records has failed to suggest any instrumental reason for these divergences .g., it is not a short-wave radiation effect, because it persists into the hours of darkness), nd it may be that this is evidence of a difference in the shapes of temperature and wind rofiles at the end of the day. If so it is a major difference in shape, and not simply a ifference in the height of the plane of zero displacement in two logarithmic profiles. may be a consequence of the small size of the test area, to be discussed later with a milar effect in the vapour-pressure profiles.

The straight lines through the top four points have been drawn to give what seemed sually the best fit, sometimes guided by the position of the fifth point, sometimes ignoring . They begin at the top point, and, in two ways, give a coarse picture of the daily cycle sensible-heat transfer. The slope, measured anti-clockwise from the vertical, is a measure upward transfer (at constant shearing stress); and a more accurate measure is obtained om the area of the right-angled triangle of which the fitted straight line is the ypotenuse. Values of heat transfer calculated in this latter way are given in § 9.

(*ii*) *Vapour pressure*

In structure, Fig. 11 is the same as Fig. 10. The daily pattern is again well represented y the general means in the bottom right-hand corner and, except for the period a, 00-04, here is always a humidity lapse, i.e., upward vapour transfer. As the slope and area of the iangle have the same significance here for evaporation as they had in Fig. 10 for sensible eat transfer, it is clear that the evaporation process is dominantly a daylight phenomenon.

Dependent as the points are on a temperature difference, the scatter is greater than n Fig. 10, as expected, and though nothing but a straight line can be fitted, it is of interest o see over what a large part of the day there is a consistent deviation from the straight ne, of the same kind as that noted for the *e* 16-20 profiles of temperature in Fig. 10. n this particular period, indeed, the estimated vapour pressure at $h + 60$ cm is nearly ways less than that at $h + 110$ cm, and this deviation from expectation corresponds to n *underestimate* of the wet-bulb temperature at the lower of the two levels. As normal ources of error in wet-bulb psychrometry (inadequate ventilation, dirt, impeded water ow, radiation) all act in the opposite sense, curative action has not been possible because ne source of the trouble has not been diagnosed. The general nature of the deviation om the straight line – and the following argument will apply to the temperature profiles oo – might be regarded as a consequence of the small size of the test area. For a fetch f 3×10^3 cm, and a wind speed of 300 cm sec^{-1}, the thickness of the turbulent boundary

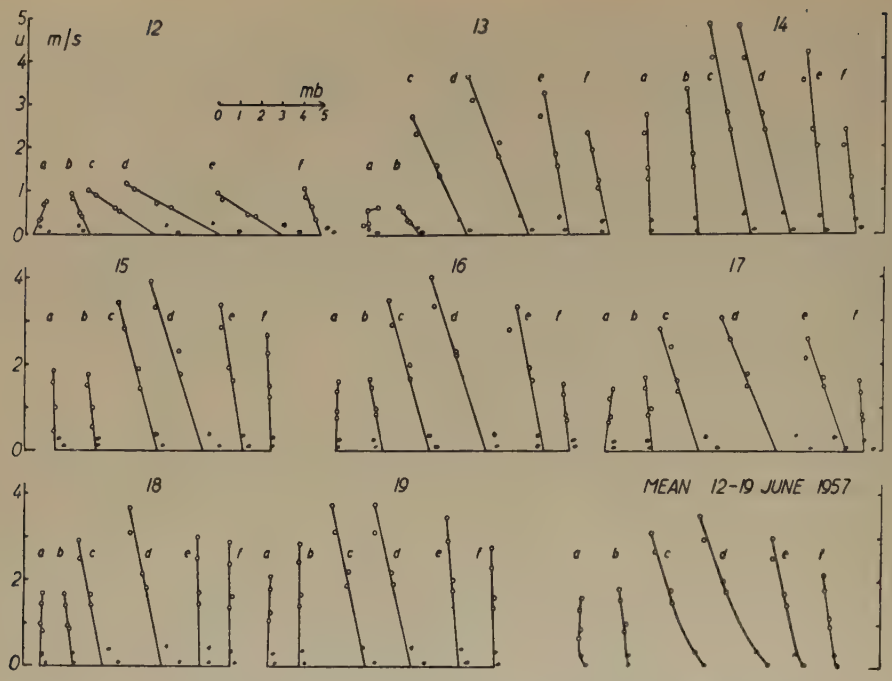


Figure 11. 4-hr mean values of wind and vapour pressure. See caption to Fig. 10.

layer over a smooth plane (Sutton 1953, p. 75) is near 50 cm, and for $z_0 + d \simeq 40$ cm, this puts the transition region at about 90 cm above ground, i.e. between the second and third levels of recording above the crop. (A more recent treatment by Elliott (1958) for a rough surface gives 700 cm as the boundary layer thickness for $z_0 = 10$ cm, a change that renders superfluous the following qualitative discussion. If Elliott's analysis is confirmed, then the plot is *not* too small). Broadly, the temperature and vapour pressure at the two upper levels will be mainly characteristic of the area upwind of the test area and not markedly dependent on the surface beneath : the values at the two lower levels, within a few centimetres of the crop, will be mainly characteristic of the crop and not markedly dependent on what has happened upwind. If the evaporation rate upwind is less than that from the test area, then in terms of Fig. 11 it is to be expected that the upper pair of points will be displaced to the left of the lower pair. At the same time there should be greater sensible-heat transfer upwind, but this too can conform to the pattern of Fig. 10 *only if there is a temperature lapse.* The explanation breaks down in an inversion, when all attempts to juggle with profiles lead to the same result : the kind of distortion in an inversion should be the mirror image of that in a lapse; and it is not. In any case there are grounds for believing that if the change of surface distorts the temperature and humidity profiles it might be expected to affect the wind profile in the same sort of way, in which event Figs. 10 and 11 would not reveal the distortions. Hence, if small size is a contributory cause of distortion, it is not the only cause.

The temperature profiles profiles of Fig. 10 could be straightened by very simple corrections. If there is a zero error of $-$ 0·05°C at $h + 60$ cm and another of $+$ 0·05°C at $h + 10$ cm, the distortion would vanish. This size of error might occur in setting up the equipment, or might develop later because of changes of electrical resistance – at a joint or a contact – of 0·018 ohms. The chart evidence shows that the zero for the wet-bulb element remains the same as for dry-bulb element, i.e., wet-bulb depressions are not affected, and complete removal of the temperature anomaly in this way would remove about one-third of the vapour-pressure anomaly. Most of the problem remains, and further speculation will be postponed until more measurements have been made over a larger area and with more detail in the profile measurements.

9. Evaporation and heat transfer 12-19 June 1957

With Table 7, Figs. 10 and 11 can be used to give estimates of heat transfer and evaporation over the period. Dr. J. L. Monteith has kindly provided the records of evaporation obtained from the large field balance set up in the middle of the area. This has a surface of 5 × 5 ft, is 2 ft. deep, and the surface is at soil-surface level. The crop growing on it was the same height as the surround (over which these meteorological measurements were taken), but there was a small gap on the east side, necessary to get access to the recording mechanism, which resulted in an excess of short-wave radiation reaching the weighed soil block during the morning. Dr. Monteith will, in due course, consider the exact magnitude of the error, and its effects; for the present it suffices to know that in the periods 04-08, 08-12, the direct estimate of transpiration will be an overestimate by perhaps 10 or 15 per cent. From the same source there are estimates of net radiation, taken over the crop.

In computing evaporation and heat transfer the best values of d and z_0 have been determined for each of the profiles in Fig. 7, and the intercepts on the $u = 0$ axis of the straight lines of Figs. 10 and 11 read off to give the appropriate values of T_0 and e_0. Only daily totals and long-period epoch means are given in Table 8, but details for three days appear later, in Table 9.

TABLE 8. Components in the energy balance 12-19 June 1956

(a) Daily totals (all in cal cm^{-2})

Day	Upward heat flux Q_s Eq. (5)	Upward vapour flux Calc. E_A Eq. (4)	Upward vapour flux Measured E_B (balance)	Calc. E_P (Penman)
12	− 54	196	273	186
13	− 36	296	347	260
14	− 8	362	360	287
15	− 19	251	323	267
16	− 13	387	333	305
17	− 34	198	282	241
18	+ 12	143	201	267
19	− 38	253	193	291
Total	− 190	2086	2312	2104
Mean	− 24	261	289	263

(b) Mean daily values (cal cm^{-2}/4 hr)

Period	Upward heat flux Q_s Eq. (5)	Upward vapour flux Calc. E_A Eq. (4)	Upward vapour flux Measured E_B (balance)
a 00-04	− 15·0	− 0·2	− 0·2
b 04-08	+ 4·5	10	24
c 08-12	+ 31·5	82	96
d 12-16	+ 13·5	106	107
e 16-20	− 25·5	49	57
f 20-24	− 33·0	14	5
Total	− 24	261	289

The extra estimate in the last column of part (a) of the table is based on standard procedure (e.g., as in Penman (1956)) except for the use of the Rothamsted radiation record where normally only ' duration of bright sunshine ' is available for use in an empirical equation. Otherwise the temperature, humidity and wind data are those taken in the meteorological enclosure as daily routine.

The table needs little commentary. Part (b) shows clearly that the major discrepancy between calculated and measured evaporation occurs when expected, between sunrise and midday when the measured value is known to be too big because of over-exposure to sunshine. The main evaporation loss takes place between 08 hr and 16 hr, and there is very little at night. The daily cycle of sensible heat exchange in part (b) puts into figures what was qualitatively obvious in Fig. 10, though it is a little surprising to find that the net daily transfer is downward on every day except 18 June, but a cursory examination of the more detailed energy-balance data in Dr. Monteith's possession (net radiation, ground-heat flux, etc.) suggests that both the sign and order of magnitude of the sensible-heat flux in Table 8 are right. In the computation that led to the last column of part (a) of Table 8 there is, by implication, an indication that on all days the estimated net sensible-heat flux was upward (and small), and it is valuable to have confirmation of what has always been recognized, namely that a formula produced for a short crop such as grass will somewhat underestimate evaporation from a rougher crop through which some air movement is possible.

The values in Table 8 have the discrepancies all too familiar in comparisons of calculated and observed evaporation, and, except favourably, they are not to be read as criticism of the validity of the aerodynamic estimate (Eq. (4)), which is made here under conditions often far removed from those to which it is applicable. It has also been applied to data much less precise than those usually sought in short-period tests or applications of the equation, and indeed the chief value of Table 8 is as a trial of the data and the equipment that produced them. Continuous recording necessarily means some loss of precision in short-period measurements, but in the absence of systematic errors in the equipment there is always the hope that long-period means will be of adequate accuracy. With one reservation this appears to have been so here, and with interpretation strained beyond the limit of designed performance the equipment has given more than was hoped for.

The reservation refers to the humidity profiles of Fig. 11. As already noted, even after taking out random errors by averaging over 28 hr, there are deviations from expectation, one of them physically absurd. The deviation will be shown in another way in the following section, whose main purpose is to reveal another aspect of the temperature and humidity relationships of the crop.

10. Daily cycle of temperature and humidity

The technique used for Figs. 10 and 11 has been applied to the third possible combination of parameters. Two consecutive days in marked contrast (18 and 19 June) are represented in Fig. 12, and in greater detail in that some hourly means are used. 18 June had the wettest atmosphere of the week : 19 June had the driest, and ended in rain. Mr. F. H. Ludlam drew our attention to the 19th as a day of anticyclonic subsidence over the district, and the figure reveals some of the interactions of micro- and macro-meteorology as the subsidence developed. The upper part carries the 48-hr record of air temperature, vapour pressure, and relative humidity at the top level of recording (the wind record is on Fig. 7, with detail in Table 5 for a lower level).

The remainder of Fig. 12 is in two parts. Two horizontal rows of diagrams relate 4-hr means of temperature and vapour pressure at the same heights; and four diagrams below show hourly means during the transitions between inversion and lapse, with absolute values of temperature (ordinate) and vapour pressure (abscissa) indicated. Though the plots of T against e show great scatter, as expected from Figs. 10 and 11, most of the groups of circles (observations above the crop) can be fitted by the straight lines that imply identity of profile shape. Where exceptions occur, here and in many other records that have been examined, there is no consistency in the form of the distortion, and the cause is still a matter for speculation. In these diagrams the positions of some of the squares (observations inside the crop) may be in error, right or left, because of an unavoidable uncertainty in psychrometer calibration at very low wind speeds; and with that warning the lower half of Fig. 12 can be left to tell its own micro-meteorological story – with one

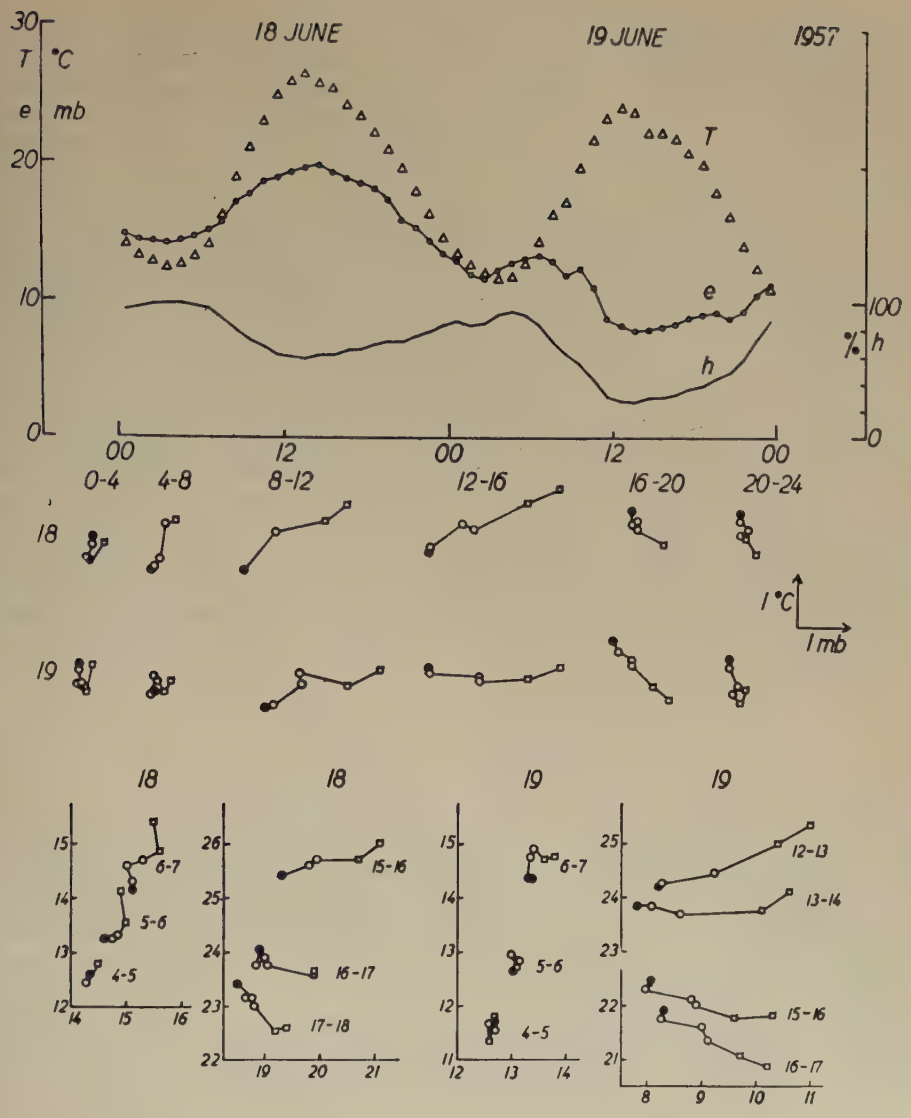


Figure 12. Temperature and humidity on ' normal ' day (18 June) and day of subsidence (19 June). Top : temperature, vapour pressure, and relative humidity at 180 cm. (Mean for the hour). Middle : 4-hr mean values of temperature (ordinate) and vapour pressure (abscissa). Full point : highest level of recording. Squares : inside crop. Bottom : 1-hr mean values of temperature and vapour pressure during transitions to lapse or inversion. Scale as for middle group.

exception. Comparing the two days after sunrise, from 05 to 07 on 18 June the hottest layer was near the ground, but on 19 June it was at the top of the crop, an unexpected contrast because there was no sunshine before 07 on the 18th, and 1·8 hr before 07 on the 19th, i.e., during the period when direct radiation could reach the soil surface most easily. With the unbroken sunshine there was, of course, a clear sky, and it may be that at this time of day it is the long-wave energy balance rather than short-wave income that determines the temperature gradients in the crop.

This long-wave radiation exchange may be dominant in the macro-meteorology also. At first sight the upper part of Fig. 12 suggests that the subsidence on 19 June has had a cooling effect, in that the maximum temperature is lower, and occurs earlier, than on the 18th. Because of the reduced vapour pressure, under a clear sky the rate of

net long-wave radiation loss near noon on the 19th would be about one and a half times that on the 18th, at about 13 compared with 9 cal cm^{-2} hr^{-1}, and this seems to be the most significant contribution of the subsidence to the local energy balance, i.e., reducing the net radiant energy flux to the crop, and with this, presumably, tending to reduce the rate of evaporation. With saturation deficits of 14 mb (18 June) and 22 mb (19 June) at midday it might be expected that evaporation rates would be greater on the 19th than on the 18th, but the field evidence indicates that they were of the same order of magnitude, and the crop somehow solved the problem of keeping its water-loss rate in step with the local energy balance without undergoing the desiccation that these vapour-pressure differences might impose. The biological aspect of this plant response is considered in the next section, but one physical aspect, such as would apply to wet green blotting paper, can be seen from Fig. 12, in the bottom right-hand diagrams. Forcing the evaporation, in the absence of a supplement to radiant energy supply, must cause cooling, and by 14 hr 19 June the top of the crop had become a sensible-heat sink. In a complex coupled system such as this it is not easy to separate driver from driven, but, broadly, it might be stated that the crop adjusts its rate of transpiration to the local energy balance, and the atmosphere adjusts its vapour-pressure gradient to the water flux and the prevailing transport constant. Assuming that there was some subsidence at crop level, could this have contributed anything to the transport mechanism ? A simple calculation for the test area shows it to be very unlikely, for to alter the aerodynamic estimate of evaporation by 10 per cent would need a downcurrent of 1 cm sec^{-1} which would produce an increased horizontal flow in the crop of about 25 cm sec^{-1}, an increase that could have been detected, but was not.

11. The source of water vapour

(i) *Inference from profiles*

Given the vapour-pressure profile and knowledge of the variation of the transport constant, the build-up of flux from bottom to top of the crop can be estimated. In many periods the vapour-pressure gradient within the crop was nearly uniform, i.e., the shape of the flux build-up curve is, in effect, the shape of the transport constant curve. As a first guess, one might say that this has the same shape as the profile of wind velocity, in which case the part of the wind velocity profile within the crop, as shown in Fig. 9, can be taken as a guide to what might be expected.

Fortunately it is possible to turn the problem around, i.e., to get a direct estimate of the build-up of flux, and then from the vapour-pressure gradients to get an estimate of the way the transport constant changes from the ground up through the crop.

(ii) *Estimation of horizontal flux from leaf to air*

The transpiration stream passes out of the leaves as vapour through stomata normally completely closed in the dark, and normally fully open in daylight, provided there is no moisture stress in the soil, when opening may be incomplete. The flux per unit area of leaf will be proportional to the vapour-pressure drop between the sub-stomatal cavity inside the epidermis and the air outside; and inversely proportional to the sum of two diffusive resistances in series, the first being that of the stomatal array and the second that of the air outside the leaf, both being conveniently expressed as effective lengths per unit area of surface. For wheat under laboratory conditions (Milthorpe and Penman, in preparation) with a stomatal population of 7,000 per cm^2, including both sides of the leaf, the maximum diffusive conductivity measured is $S_D \simeq 3{\cdot}0$ cm^{-1}, i.e., $1{\cdot}5$ cm^{-1} per side, giving an effective length, per side, of $1/1{\cdot}5 = 0{\cdot}67$ cm. (In the field the stomata may open wider, i.e., this is probably an upper limit of resistance in full illumination). For the external resistance de Wit (1958) quotes results of experiments under comparable conditions by de Vries and Venema (1954) leading to an external length, L_a, as $L_a = 3{\cdot}26\ u^{-0{\cdot}70}$ cm, where u is the wind speed over the leaf in cm sec^{-1}. This expression

may give too big a value at low wind speeds, for Maskell's (1928) data suggest a value $L_a \simeq 1$ cm in still air (see more detailed discussion in Penman and Schofield (1951)), but it will suffice for this probing phase in the analysis. The ' effective length ' per unit area of leaf surface with fully open stomata will be

$$L = 0{\cdot}67 + 3{\cdot}26\, u^{-0{\cdot}70} \text{ cm} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

having a range from about 4 cm at $u = 1$ cm sec^{-1} to 0·73 cm at $u = 300$ cm sec^{-1}.

If e_0 and e are the vapour pressures inside and outside the leaf, the flux per unit area of surface is proportional to $D\,(e_0 - e)/L$ where D is the molecular coefficient of diffusion of water vapour in air, to be taken as 0·24 cm^2 sec^{-1}. Putting in the appropriate constants, and expressing the flux in calories to facilitate comparison with Table 8, the flux is

$$\delta E = 0{\cdot}38\,(e_0 - e)/L \text{ cal hr}^{-1} \text{ per unit area of leaf.} \quad . \quad . \quad (7)$$

Suppose that the leaf area in any layer between z and $z + 1$ cm is a times the area of ground the crop occupies; then the transpiring surface of leaf, per unit area of ground, between levels z and $z + \delta z$ is $2a\,\delta z$, and $\int_{h/4}^{h} a\,dz = \lambda$, where λ known as the ' leaf area index ' (of order unity), is a measure of the fraction of ground area that would be covered if all the leaves of the crop were cut off and laid on the ground without overlapping before the first layer was complete. Hence the horizontal vapour flux into the layer of air between levels z and $z + \delta z$ is

$$\Delta E = 0{\cdot}38\,(e_0 - e) \cdot 2a\,\delta z/L \text{ cal hr}^{-1} \text{ per unit area of ground,}$$

and the upward flux at any height z within the crop is

$$Ez = \int_{h/4}^{z} \Delta E,$$

assuming there are no leaves below $z = h/4$.

To make progress, an assumption about a is necessary. From $z = h/4$ the leaf density increases fairly rapidly, and it has been assumed that this increase is uniform up to $z = h/2$ and the density then remains constant between $z = h/2$ and $z = h$. The rest is simple geometry; with an added minor complication on all but 12 June. As in § 7, it has been assumed – arbitrarily – that the wind depresses the top of the crop by about 10 cm, with maximum effect in the top layer of the crop.

To determine the ' effective length', L, it is necessary to know how u varies with height within the crop, and the curves of figure 9 will be used to give the shapes of wind profiles, treated as ' calm ' for 12 June, and ' windy ' on 16 and 19 June.

To determine $e_0 - e$, the available profiles of vapour pressure in the crop give e, and it remains to determine e_0. This will be assumed to be the saturation vapour pressure at leaf temperature. Leaf temperature was measured at $z = 55$ cm by an element with a leaf lightly folded round it; and at $z = 22$ cm by an element pushed into the junction of a leaf with the main stem. Throughout most of the eight days 12-19 June these temperatures were less than the estimated air temperature at the same level, the range of difference being from − 2·1°C (8-12 on 12 June) to + 0·3°C (16-20 on 19 June), suggesting that it is the ground surface rather than the crop that dominates the main energy exchanges with the atmosphere. (Perhaps this is not so surprising, as the thermal capacity of the whole crop is no more than that of a 2-3 mm layer of soil). After tedious drawing and study of many curves, a welcome simplification emerged : within 10 per cent, during daylight, the saturation deficit $e_0 - e$ (where e_0 is for leaf temperature) is uniform throughout the crop, and the mean of the two values that are directly determinable is entirely adequate for the present purpose.

For each of the three days 12, 16 and 19 June, six values of $e_0 - e$ have been determined for the six 4-hr periods, mean values of u determined for 10 cm intervals of z from bottom of leaf layer to the top of the crop and thence values of L read from a graph of Eq. (6).

The best available estimate of the leaf area index during the week was $\lambda \simeq 2$: it may have increased by 20 per cent during the week, but as there is an uncertainty of at least 0·5 in the visual estimate the value $\lambda = 2$ will be used for all days. The horizontal transfer in each layer was then calculated and summed from bottom to top, the total being the estimated vapour flux leaving the crop, i.e., the same quantity as appears in Table 8. For two days this is true without qualification, for on both 16 and 19 June it is safe to assume that there was negligible vapour movement between $z = 0$ and $z = h/4$ – the soil surface was thoroughly dry. On 12 June, however, the soil surface was wet, and values of the flux from the soil have been calculated from Eq. (7) using as e_0 the saturation vapour pressure at mean soil-surface temperature, and as e the vapour pressure measured at 10 cm above the ground. The wind velocity at the lowest observation level ($z = 25$ cm) has been used to determine L. (There is no stomatal factor here). Table 9 shows the estimates, by periods, for each of the three days, including the direct measurement by balance, E_B, the aerodynamic value calculated from Eq. (4), E_A, and the new estimate, E_C, with its components E_L from the leaves, and E_S from the soil for 12 June. In the daily totals the new estimate (soil loss apart) has been restricted to daylight hours on the assumption that the stomata were closed at night.

TABLE 9. ESTIMATES OF EVAPORATION 12, 16 AND 19 JUNE 1957 (cal cm^{-2} in 4 hr)

Date	12					16			19		
period	E_B	E_A	E_L	E_S	E_C	E_B	E_A	E_C	E_B	E_A	E_C
0-4	0	− 4	− 2	6	4	− 1	− 2	2	0	2	10
4-8	19	9	4	6	10	26	13	13	17	6	20
8-12	87	32	30	18	48	110	122	98	69	82	101
12-16	111	66	57	41	98	130	163	182	69	113	150
16-20	52	68	25	22	47	60	83	124	38	40	116
20-24	4	25	− 3	6	3	8	7	33	Rain	11	36
Comparable totals	273	197			203 + 12	333	386	417	193 +	254	387

The agreements, both in periods and daily totals for 12 and 16 June are extraordinarily good – far better than was expected after all the assumptions and approximations that have been imposed – and it may be safe to consider the layer-to-layer build-up of the flux on which the estimates of E_C are based as probably about right. Similarly, it may not be vain to look for possible causes of the major discrepancies on 19 June, and these are conveniently taken first.

In the eight days 12-19 June the total water loss (Table 8) was equivalent to 40 mm of rain and the soil-moisture deficit had reached about 100 mm, and there is little doubt that by 19 June the greatly reduced hydraulic conductivity of the soil was making it difficult for soil water to move quickly enough to the plant roots to meet the maximum demand of the transpiration load. This is most certain in the balance with its limited depth of soil, and the check in transpiration rate in the period 12-16 on 19 June – comparing E_B and E_A – probably indicates a major difference between the crop on the balance and the main area round the balance. Here the story is probably as follows. During the early forenoon the transpiration rate would be at the rate E_C, the water coming mainly through the roots from the soil, with a little from the leaf tissue. As the load increased the poor conductivity of the soil caused a greater desiccation of the leaf tissue, a loss of turgor in the leaves, and partial closure of the stomata, i.e., the quantity L increased and the transpiration rate decreased. This self-protective mechanism of the leaves has an

inefficient aspect, for the lowered transpiration carries with it the implication of an increased leaf temperature, i.e., an increase in e_0 (and a possible decrease in the external vapour pressure e). Hence when the computation of E_C begins to go wrong because the assumption of fully open stomata is not satisfied, the value of E_C not only exceeds the actual transpiration rate but also exceeds the potential transpiration rate to be expected if the stomata were fully open. When the error in E_C becomes positive it is likely to be greatly so, and it is fairly certain that from mid-morning to sunset on 19 June transpiration was checked by stomatal closure. As the stomata are the channels for uptake of carbon dioxide, growth would be checked too.

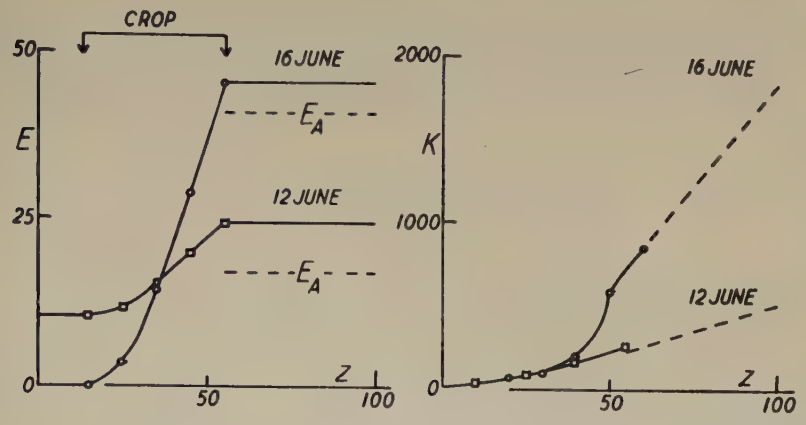


Figure 13. Left : estimated upward vapour flux (cal cm^{-2}/4 hr) on 12 June (12-16, soil surface wet) and 16 June (12-16, soil surface dry) calculated from assumed leaf density, stomatal conductivity, saturation deficit and ventilation (full line joining points). Aerodynamic estimate above crop level shown by dashed line. Right : estimated values of transport constant $cm^2\ sec^{-1}$. Full line : from estimated E and slope of vapour-pressure profile. Dashed line : from wind velocity profile above crop.

(iii) *The build-up of vapour flux*

Out of Table 9, two periods have been chosen to show the way in which the totals of E_C have been built up layer by layer. Data from 19 June have not been considered for reasons obvious from the preceding paragraph : those to be used are for the period 12-16 on 12 and 16 June, and provide two contrasts, namely between calm and windy, and between wet-surface soil and dry-surface soil. Fig. 13 (left) shows the result with the ordinate in cal $cm^2\ hr^{-1}$, and the indicated crop thickness – the same for both periods – is the nominal thickness for 12 June ($h = 55$ cm), and the compressed thickness for 16 June ($h = 65$ cm). Both curves have the same sort of shape within the crop, and the departure from linearity near the bottom is determined almost entirely by the assumed pattern of leaf density. For this crop in these periods all leaves seem to contribute about equally to the transpiration stream. The other features of the left-hand part of the figure are already in Table 9, namely that on 12 June about half of the flux came from the soil, and that for both days the aerodynamic estimate is smaller than the estimated crop transpiration.

(iv) *Variation of K with height*

While some reservation about accuracy is desirable for the preceding paragraph, it is essential here : the present purpose is to establish possible trends, and no more. Through somewhat scattered points, smooth curves of e against z were drawn, adding on

12 June a value at $z = 0$ representing the saturation vapour pressure at soil surface temperature. After this subjective operation, slopes were measured at a few values of z – another subjective operation – and from the flux at these values, read from the left-hand diagram, the values of K ($cm^2\ sec^{-1}$) were computed. These are plotted in the right-hand diagram, and though those within the crop could easily be wrong by a factor of two or three, it appears that in the lower part of the crop they are not appreciably different on the two occasions. On the calm day (12 June) there is a uniform increase with height over the range of computation, represented by the full line. The dashed line, starting at the height at which the full line ends, gives the value of K computed from the wind profile for the period, and though there is a discontinuity on 12 June, on 16 June there is complete continuity.

This apparent internal consistency suggests that the shape of the K-curve for 16 June may be correct, i.e., it resembles the U-curve of Fig. 9 on a windy day. Combining the two it is obvious that the product $K\,\partial u/\partial z$ must increase exceedingly rapidly near the top of the crop, or, looked at from above, there is a very rapid absorption of momentum at the top of the crop.

12. Discussion

Biological. In his inaugural lecture at the University of Nottingham in 1955, Professor F. L. Milthorpe stated :

> 'We can understand the association between the plant and the environment in which it grows only if we know both intimately. In the past these two fields of knowledge have been too widely divorced from each other – the biologist has recoiled in horror from the very thought of attempting to understand the physics of the lower atmosphere, and the micro-meteorologist has sadly neglected some of the more important principles of plant growth.'

The intimate physical knowledge which this paper attempts to supply for one crop can be considered with two of the ways in which crop yields might be increased, namely, (i) more efficient use of healthy plants; and (ii) preventive action to maintain healthy plants. The 'important principle of plant growth' needed here is that the process is one of assimilation of carbon dioxide, and the physics of the process must have much in common with that of the transpiration process discussed in § 11, with the same two spatial divisions. Above the crop we expect a vertical flux (downward) independent of height : within the crop we expect the horizontal flux to vary with height, depending on the concentration gradient and on a transport constant which should be of the same order as, if not identical with, that of the transport constant for water vapour. It appears from § 11 that this constant is a fairly conservative quantity over a wide range of external wind speeds, because the wheat crop (and presumably other cereals too) appears to be self-sealing so that inside the canopy mixing processes are always relatively slow, a conclusion expressed quantitatively in Fig. 13 and Tables 2, 3 and 4, and supported qualitatively by the persistence of hot spots and damp spots (Fig. 6 and Table 6). Given better measures of this transport constant, and some measure of carbon-dioxide gradients within the canopy (a rather remote possibility, as yet), it might then be possible to draw an analogue to the left-hand diagram of Fig. 13, with net assimilation rate as ordinate, a diagram which would show if all the leaves are contributing to growth. The possibility that some leaves may be parasitic on the rest of the plant arises from the work of Watson (1958) with kale, for he has shown that selective defoliation might well increase the total dry matter produced in a complete growing season. This is a challenge to the meteorologist to get the micro-physics of the assimilation process clarified; and beyond it there may be a challenge to pass on to the plant breeder.

The possibilities of preventive action are very great, but are very much less certain because we do not know what micro-weather information is needed. The rusts, the mildews, the blights and the scabs; the insect pests, either as parasites or vectors for

disease, all have their own optimum range of micro-climate, but it is a rare event for a disease study to incorporate meteorological measurements, and there was no (obvious) disease on any of our three wheat crops. Generalizing, it is clear that any migrant insect that prefers calm conditions for flight would have found them at all times in the 1957 wheat crop. On sunny days the same insect would have a wide range of temperature to choose from, and on the first sunny day after rain a wide range of humidities, both absolute and relative. The chief weather requirement of many plant diseases is a high relative humidity, and for some, at one stage in their life cycle, a film of liquid water on the leaf. Although dew formation on wheat is dropwise, on this scale even a 1 mm diameter drop is a fairly large swimming bath covering perhaps 30 stomata, so there are plenty of openings available for those fungi that infect through the stomata. But fungi may attack the stem too, and, though it is not always present, such an attack is very frequently the precursor to the flattening of wheat crops during July storms. There may be something useful to be learned from a detailed study of the effect of the infection on the rigidity of the stalk, and hence on the natural period of vibration of the plant when in full ear and wet, and whether or not there can be damaging vibration forced by turbulent eddies of the right frequency.

Less than 5 per cent of the available measurements have been used in this survey : the rest go where they are intended to go, into storage against the day when the biological requirements become clearer and the questions more precise.

Physical. Discussion can be very brief, as most items have been discussed in the course of writing. There are some anomalies, clearly displayed, which need further thought and further experimental work : there are some places where the sole achievement is in giving quantities where the qualitative behaviour could be confidently predicted, as, for example, in the general shapes of temperature and humidity profiles by night and by day, and the effect of plant density on them. As a supplement to the section on dew, it might be noted that, on all occasions of observed dew, the gradients of humidity, either from the standard profile or from the special recorder, have shown a humidity lapse in the crop and a humidity inversion above it : on no occasion has dew been observed when there was a humidity lapse above the crop, an event that can occur over very short grass (Monteith 1957).

The evidence acquired and presented has not been good enough to affect the arguments about identity of profile shapes for wind, temperature and humidity : but, at worst, the assumed identity is a very good first approximation that can be used, as in § 9, to compute transport rates over periods considerably longer than those used, for example, by Rider (1957). More data will be examined, choosing occasions when Dr. Monteith's records of energy exchanges are as complete as possible, and it is hoped that the examination will confirm the impression obtained here : that continuously recording anemometers and psychrometers at four levels above the crop can be accurate enough to give at least weekly (if not daily) totals of transpiration. Reduction of the data would be tedious, but not impossibly so, and there are some types of field experiment for which the effort would be worth while.

One other aspect of the results is worth comment. The existence and persistence of hot spots implies the possibility of small local convective activity, and though the analysis of § 10 suggests that over the plot as a whole such activity has no detectable effect on water-vapour transfer, on a small scale it may determine where airborne pollution will reach the crop, and influence the early movement of any insects or spores leaving the crop.

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A theory of fronts in relation to surface stress

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Summary

Surface-stress terms are introduced into the equations governing the motion of the cold air ahead of a warm front or behind a cold front. Solutions for the steady state indicate that a katafront inversion tends to become horizontal at increasing distances from the front whereas an anafront inversion *may* continue to slope upwards indefinitely. The depth of cold air will, when the upper flow is curved cyclonically, actually decrease with increasing distance from a cold katafront. It is also shown that the existence of a sharp surface warm front depends on the sudden breakdown of a trailing layer of cold air. The thickness of this layer increases with decrease in latitude and as a result, sharp warm fronts are unlikely to occur at low latitudes. The conclusions concerning cold fronts are in broad agreement with Sansom's (1951) observations.

1. Introduction

In recent years theoretical dynamic meteorologists have been mainly concerned with continuously varying fields. The distribution of vorticity, divergence and vertical motion and their relation to synoptic development have been studied with some success. With the exception of the study of pressure jumps there has been scarcely any work on the dynamics of atmospheric discontinuities. The aim of much of this recent research is the development of a successful model for numerical prediction with the aid of high-speed electronic digital computers. Fronts now receive little emphasis, ' tending to be regarded as a matter of detailed structure not essential to the general baroclinic development ' (Sutcliffe 1952). This change in attitude since the ' front-happy ' years between the wars is also reflected in recent text-books. For example, the *Compendium of Meteorology* edited by Malone (1951), does not discuss fronts in their own right – they are merely mentioned in articles on other topics. The new edition of Petterssen's book *Weather analysis and forecasting* (1956), though larger than the previous edition (1940), has less space devoted to fronts. Many examples of other aspects of this trend could be given.

It still remains a fact, however, that in middle latitudes a high percentage of significant weather is associated with fronts and it is therefore essential for their occurrence and movement to be forecast correctly. Even if modern methods of numerical prediction eventually become capable of giving accurate forecasts of the large-scale flow patterns it will still be necessary to insert fronts and to have a knowledge of the way in which they move in relation to the upper flow.

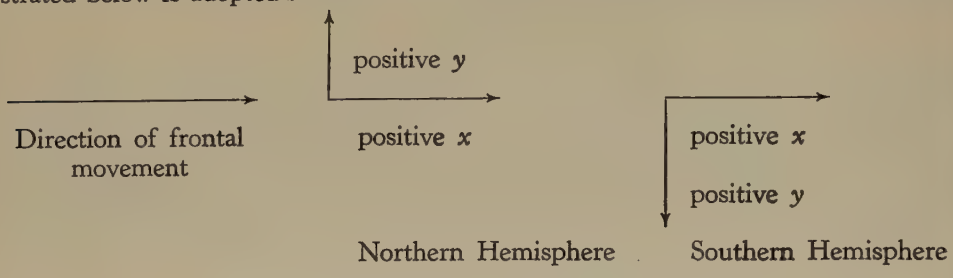
Frontal discontinuities and the organized vertical motions associated with them seem to be particularly well developed in the lower troposphere. Model experiments with rotating dishpans also indicate frontal features almost exclusively in the lower layers. This suggests that fronts are generated by surface friction acting on the baroclinic motions of the lower troposphere, or at least, that surface friction plays an important part in the structure, movement and development of a front. As a result of these considerations it was thought that the dynamics of fronts could best be studied by including the effect of surface friction.

The very simple friction model adopted here leads to a surprising variety of conclusions, many of which one would think could be readily compared with available data. However, in spite of the large number of published works on fronts there appear to be very few published statistics of frontal characteristics. Also in much of the earlier work it is difficult to draw the line between conclusions that are based on observational experience and those that are the result of speculation.

It must be emphasized that the following theory depends on a large number of restrictive assumptions and cannot by any means be regarded as a complete theory of fronts. It is hoped, however, that some interest will be rekindled resulting not only in further theoretical development but also in the publication of observations and statistics of frontal characteristics.

2. Notation

In order that the equations should be applicable to both hemispheres the convention illustrated below is adopted :



(The x direction is not necessarily eastward)

The following symbols are used :

$A = F/\rho V$

A_0, A_1, A_2 = values of A when $v = 0, v_1, v_2$ respectively

c = speed of frontal movement

f = Coriolis parameter

F = surface friction (a function of V)

F_x, F_y = x and y components of F

$\overset{*}{g}$ = modified gravitational acceleration $\left(\frac{\rho - \rho'}{\rho}\right) g$

h = height of frontal inversion (depth of cold air)

$h_1 = A_1 (v_g - \Delta^{\frac{1}{2}})/2 f (u_g - c)$

$h_2 = A_2 (v_g + \Delta^{\frac{1}{2}})/2 f (u_g - c)$

u, v = x and y components of the wind in the cold air

u_g, v_g = x and y components of the geostrophic wind in the warm air

$v_1 = (v_g - \Delta^{\frac{1}{2}})/2$

$v_2 = (v_g + \Delta^{\frac{1}{2}})/2$

V = wind speed in the cold air

V_g = geostrophic wind speed in the warm air

Δ = the discriminant given by $v_g^2 - 4c(c - u_g)$

ρ = the density of the cold air

ρ' = the density of the warm air

$\eta = v_g/c$

$\xi = u_g/c$

3. Assumptions

It is the purpose of this paper to determine the effect, *per se*, of surface friction on the shape and characteristics of a frontal surface. The front is considered to be in quasi-equilibrium, i.e., the effects of development and acceleration of the front are not considered. The following assumptions are made :

(i) The air is dry and incompressible.

(ii) The front is a perfect discontinuity of density across which no mass flow takes place.

(iii) The front moves in the positive x direction at a constant speed without change of shape.

(iv) The pressure gradient in the warm air is taken as a known constant and for convenience is expressed as a geostrophic wind with components along and perpendicular to the front.

(v) Variations in the y-direction are negligible except for the pressure variation implied by assumption (iv).

(vi) Vertical accelerations are negligible.

(vii) The frictional coupling between warm and cold air is negligible.

(viii) Surface friction acts in a direction opposite to the wind in the cold air and is a function of the wind speed.

Some of these assumptions can be partly relaxed without a serious increase in the complexity of the theory. For the present we shall be content to keep the theory as simple as possible.

In the nose of the cold front, i.e., in the strip a few kilometres wide behind the front, assumption (vi) is not usually realistic. The comparatively stagnant warm surface air will in most cases be pushed upwards as the cold front penetrates (regardless of whether the front is of kata-type or ana-type) producing vertical accelerations and circulations in both warm and cold air. These effects will, in general, accentuate the nose and cause various phenomena of the type considered by Berson (1958). The characteristics of the nose are often further modified by the development of thunderstorm cells in which condensation plays a leading role.

It is apparent from the preceding remarks that the following theory will not usually apply to the frontal nose. It is expected that in the absence of both rapid development and appreciable variation along the front (i.e. in the y-direction) the theory will generally give at least a qualitative picture of the fields of motion and pressure in the region extending from some 10 to 50 km to a few hundred kilometres behind a cold front, and the region extending a few hundred kilometres ahead of a warm front. The spatial variation of the geostrophic wind in the warm air is an important factor determining the upper limit of the region of applicability of the theory. It is comparatively easy to modify the theory to take account of variations of the y-component of the geostrophic wind but less easy to include variations of the x-component. The theory in its simplest form will not apply to situations with strongly curved upper flow and has only limited application when the curvature is moderate. The effect of slight curvature of the upper flow on katafronts is considered in Section 7.

4. Basic equations

Since the frontal inversion is assumed to be a boundary across which no mass flow takes place, the equation of continuity can be written :

$$\left.\begin{aligned} &\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}\int_0^h u\,dz=0 \\ \text{or}\quad &\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(h\,\bar{u}) \quad =0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

where the superscript bar denotes the mean with respect to height in the cold air.

The sloping frontal inversion imposes a pressure gradient $\overset{*}{g}\rho\,\partial h/\partial x$ in the x-direction which must be added (vectorially) to the pressure gradient in the warm air to determine the total pressure gradient in the cold air. Consideration of the momentum changes and forces acting on a column of the cold air then leads to the following equations of motion :

$$\frac{\partial}{\partial t}(h\bar{u})+\frac{\partial}{\partial x}(h\,\overline{u^2})=hf(\bar{v}-v_g)-h\overset{*}{g}\frac{\partial h}{\partial x}-\frac{1}{\rho}F_x \quad . \quad . \quad (2)$$

$$\frac{\partial}{\partial t}(h\bar{v})+\frac{\partial}{\partial x}(h\,\overline{uv})=-hf(\bar{u}-u_g)-\frac{1}{\rho}F_y \quad . \quad . \quad . \quad (3)$$

The column as a whole has been considered, consequently internal frictional forces resulting from vertical shear, do not appear in these equations. Now if $u=\bar{u}+u'$ then :

$$\overline{u^2}=\bar{u}^2+\overline{u'^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\overline{uv}=\bar{u}\,\bar{v}+\overline{u'\,v'} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

whence

$$h\frac{\partial\bar{u}}{\partial t}+h\bar{u}\frac{\partial\bar{u}}{\partial x}+\frac{\partial}{\partial x}(h\,\overline{u'^2})=hf(\bar{v}-v_g)-h\overset{*}{g}\frac{\partial h}{\partial x}-\frac{1}{\rho}F_x \quad . \quad . \quad (6)$$

and

$$h\frac{\partial\bar{v}}{\partial t}+h\bar{u}\frac{\partial\bar{v}}{\partial x}+\frac{\partial}{\partial x}(h\,\overline{u'\,v'})=-hf(\bar{u}-u_g)-\frac{1}{\rho}F_y \quad . \quad . \quad (7)$$

It is difficult to assess the magnitudes of the gradients of the primed quantities. We can, however, readily estimate the ratio of the terms $\overline{u'^2}\,\partial h/\partial x$ and $h\overset{*}{g}\,\partial h/\partial x$ which appear in Eq. (6). Double-theodolite pilot-balloon ascents made at Renmark, South Australia, gave values between 0·3 and 7·0 per cent for this ratio with a mean of 2·9 per cent. The ratio of the terms $\overline{u'\,v'}\,\partial h/\partial x$ and $hf(\bar{u}-u_g)$ which appear in Eq. (7), has also been estimated giving values between zero and 8·1 per cent with a mean of 2·6 per cent. It must be remembered that near the frontal nose, where $\partial h/\partial x$ is large and h may be small, the term $\overline{u'\,v'}\,\partial h/\partial x$ is almost certainly of importance, although soundings analysed to date have not shown this. The Renmark soundings and frontal cross-sections deduced from them are discussed in Section (8).

On the basis of the above evidence it is assumed throughout the remainder of the paper that the primed terms on the left-hand sides of the equations are negligible compared with the terms on the right. The superscript bar is then superfluous and will henceforth be omitted.

If the system moves without change of shape at a constant velocity c in the x-direction then $\partial/\partial t\equiv -c\,\partial/\partial x$ and the equations can be written in terms of one independent variable. The derivatives will therefore be expressed as total derivatives d/dx. The equations then become :

$$\frac{d}{dx} h(u-c) = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$h(u-c)\frac{du}{dx} = hf(v-v_g) - h\overset{*}{g}\frac{dh}{dx} - \frac{1}{\rho}F_x \quad . \quad . \quad (9)$$

$$h(u-c)\frac{dv}{dx} = -hf(u-u_g) - \frac{1}{\rho}F_y \quad . \quad . \quad . \quad (10)$$

Eq. (8) can be integrated at once to give

$$h(u-c) = Q \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where Q is a constant representing the mass flow beneath the inversion relative to the motion of the system. In the general problem of the motion of an inversion surface, Q is not zero. Here we are concerned with an inversion that intersects the ground, i.e., a front, and ideally Q must be zero. In fact there is evidence that there is mass flow across a cold-front nose (Berson 1958) and one would expect mass flow across a warm front for reasons that will become apparent subsequently. Berson's observations do suggest, however, that Q is sufficiently small for the acceleration terms on the left-hand sides of Eqs. (9) and (10) to be negligible. We also have $F_x = \rho uA$ and $F_y = \rho vA$ where A is *not necessarily constant* but is a *positive* function of V. Whence

$$u = c \quad . \quad . \quad . \quad . \quad (12)$$

$$hf(v-v_g) - h\overset{*}{g}\frac{dh}{dx} = Ac \quad . \quad . \quad . \quad (13)$$

$$-hf(c-u_g) = Av \quad . \quad . \quad . \quad (14)$$

If the speed of movement of a cold front is greater than the x-component of the geostrophic wind in the warm air, i.e., $c > u_g$, then there will be a tendency for upsliding motion of the warm air and the front will be an anafront. Conversely, if $u_g > c$, the front will be a katafront (opposite conditions apply at warm fronts). We have adopted the ana- kata- terminology advocated by Bergeron (1937) and used by Sansom (1951). The geostrophic wind in the warm air has here been compared with the actual wind in the cold air, and as a result there is some doubt about this method of classification into ana- and katafronts particularly when the differences in speed are small. For the purposes of this classification the wind in the warm air is taken to be geostrophic since the isobars are assumed to be straight and the frontal inversion is regarded as a lid which limits the upward extent of turbulent mixing. These remarks do not apply to the nose of a cold front where the normal component of the surface wind in the warm air ahead of the front is dominated by surface friction and is often markedly ageostrophic. Consequently, as remarked previously, the warm air will usually be rising near the nose.

If the sign $u_g - c$ is adopted as a means of distinguishing between ana- and katafronts an interesting deduction follows immediately from Eq. (14). Since h, A and f are positive, v and $u_g - c$ must have the same sign. Therefore the parallel component of the *actual* wind behind a cold front is positive after a katafront and negative after an anafront. The converse holds for warm fronts; thus a positive parallel component in the cold air ahead of a warm front is associated with an anafront.

Elimination of h from Eqs. (13) and (14) gives an equation with v as the only dependent variable:

$$\overset{*}{g}v\frac{d}{dx}(Av) = f^2[v(v-v_g) + c(c-u_g)](u_g - c) \quad . \quad . \quad (15)$$

This is an important equation on which most of the following discussion is based. We shall see later that the nature of the quadratic in v which occurs on the right-hand side of

Eq. (15) is of great importance in determining the character of the solution. When $c = u_g$, Eq. (15) is trivial and the solution for h is derived by solving Eq. (13) with v identically equal to zero (see Appendix). Throughout the remainder of the paper it is assumed that $c \neq u_g$ unless the contrary is specifically stated.

5. Properties of the solutions

The present theory is not expected to give an exact quantitative description of frontal behaviour, but, subject to the limitations implied by the assumptions, should give a qualitative description together with the order of magnitude of the quantities involved. It seems unnecessary in the first instance to attempt a precise analytical or numerical solution of the equations if the relevant features of the solutions can be deduced by other means. It is indicated below how the form of the solutions can be deduced in some detail without actually solving the equations. In order to do this we must consider, among other things, the behaviour of the solution in the region where v is small, even though we know that the equation is often invalid in this region.

Where h and v are small, Eq. (15) becomes :

$$A_0 \overset{*}{g} v \frac{dv}{dx} = - cf^2 (u_g - c)^2 . \quad . \quad . \quad . \quad (16)$$

The quantity $v(v - v_g)$ has been neglected in comparison with $c(c - u_g)$ and it has been assumed, for the moment, that A is constant or varies slowly, i.e., $Adv \gg vdA$. If the origin of x is taken at the surface front the solutions for h and v are :

$$\tfrac{1}{2}v^2 = - \frac{f^2 c (c - u_g)^2 x}{A_0 \overset{*}{g}} \quad . \quad . \quad . \quad . \quad (17)$$

$$\tfrac{1}{2}h^2 = - \frac{A_0 cx}{\overset{*}{g}} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Thus in this region the frontal inversion always has a parabolic form and the negative sign in each of these equations implies that the front is always cold. This model does not give clear-cut surface warm fronts. We see later that there is always a trailing sheet of cold air in the warm front solutions and the frontal inversion ideally never intersects the ground. The model therefore appears to include the basic explanation of the relative sharpness, at the surface, of cold as compared with warm fronts.

If it is assumed that Eq. (15) is correct, then the approximate range of validity of Eqs. (17) and (18) can be seen as follows : suppose $v - v_g \simeq c - u_g$ and $c \simeq 10$ m sec^{-1}, then $v(v - v_g)$ will be about one-fifth of $c(c - u_g)$, if v is about 2 m sec^{-1}. Let us take $f = 10^{-4}$ sec^{-1}, $A_0 = 4 \times 10^{-2}$ m sec^{-1}, $\overset{*}{g} = 10^{-1}$ m sec^{-2}, and $|c - u_g| = 2$ m sec^{-1}, then the condition that v cannot exceed 2 m sec^{-1} becomes on substitution in Eq. (17) $|x| < 20$ km and $h < 400$ m.

Eq. (16) is incorrect if c is zero. The solution in this case can be found by putting $c = u = 0$ in Eqs. (13) and (14) giving

$$h = - \frac{f v_g x}{\overset{*}{g}} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$v^2 = - \frac{f^2 u_g v_g x}{A_0 \overset{*}{g}} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The slope of a stationary frontal surface near its intersection with the ground is independent of surface friction as is evident from Eq. (13).

Returning to Eq. (15) and making the reasonable assumption that the function A is a *positive monotonic increasing function of* V (and hence of v), we find that the precise form of A does not significantly influence the qualitative behaviour of the solutions. The form of

The solution depends mainly on whether the quadratic in v on the right-hand side has real factors or not. If it does have real factors, then the solution drawn as a graph of h has two horizontal asymptotes which are of interest if they occur at positive values of h. The discriminant of the quadratic indicates the existence and position of the asymptotes and is therefore an important quantity determining the character of the solution. The discriminant is defined by the equation :

$$\Delta = v_g^2 - 4c\,(c - u_g) \qquad . \qquad . \qquad . \qquad . \qquad (21)$$

It is noteworthy that Δ is independent of surface friction, considering that the theory is essentially based on the effect of surface friction. When $\Delta > 0$ the factors are real and the asymptotic values of v are given by :

$$v_1 = \frac{v_g - \Delta^{\frac{1}{2}}}{2} \qquad v_2 = \frac{v_g + \Delta^{\frac{1}{2}}}{2} \qquad . \qquad . \qquad . \qquad (22)$$

and the corresponding values of h from Eq. (14) are :

$$h_1 = \frac{A_1\,(v_g - \Delta^{\frac{1}{2}})}{2f\,(u_g - c)} \qquad h_2 = \frac{A_2\,(v_g + \Delta^{\frac{1}{2}})}{2f\,(u_g - c)} \qquad . \qquad . \qquad . \qquad (23)$$

where A_1 and A_2 are the values of the function A when v equals v_1 and v_2 respectively.

On eliminating A from Eqs. (13) and (14) and differentiating we observe that :

$$\frac{d^2 h}{dx^2} = \frac{f}{g v^2}\frac{dv}{dx}(v^2 - c\,(c - u_g)) \qquad . \qquad . \qquad . \qquad (24)$$

consequently the cold anafront and the warm katafront have points of inflexion where :

$$v^2 = c\,(c - u_g)$$

This implies a possible change from anticyclonic to cyclonic curvature of the isobars behind a cold anafront or ahead of a warm katafront. The values of h and dh/dx at the point of inflexion are given by :

$$h = +\frac{A}{f}\left(\frac{c}{c - u_g}\right)^{\frac{1}{2}} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (26)$$

$$\frac{dh}{dx} = -\frac{f}{g}(v_g + 2\,[c\,(c - u_g)]^{\frac{1}{2}}) \qquad . \qquad . \qquad . \qquad (27)$$

showing that dh/dx is about 10^{-2} and h is of the order of 500 m, (typical values of A/f are given in Table 1).

TABLE 1. VALUES OF A/f IN TENS OF METRES

Latitude	$10^5 f$ sec^{-1}	A cm sec^{-1}		
$\Phi°$		2	4	8
20	5·0	40	80	160
40	9·5	21	42	84
60	12·6	16	32	64

6. CLASSIFICATION OF THE SOLUTIONS

It is clear from the preceding section that solutions of quite different characteristics will arise according to the values of the significant parameters. It is a matter of some difficulty at this stage to decide on the best form of classification to propose. From a purely formal or mathematical point of view the classification requires to be succinct,

non-overlapping and exhaustive. However, a classification made on this basis is not necessarily best aligned to questions of practical meteorological relevance. In illustration of the point it is to be noted that no formal distinction has been, or requires to be, made between warm and cold fronts. This is a purely consequential distinction to be determined as the solution for h slopes upwards or downwards in the direction of motion of the front; yet according to meteorological requirements it is in many ways the most important attribute in any classification system.

From the formal point of view (neglecting special cases) there are only four types of solution depending on whether h_1 and h_2 are imaginary, both negative, both positive or of opposite sign (see Fig. 1).

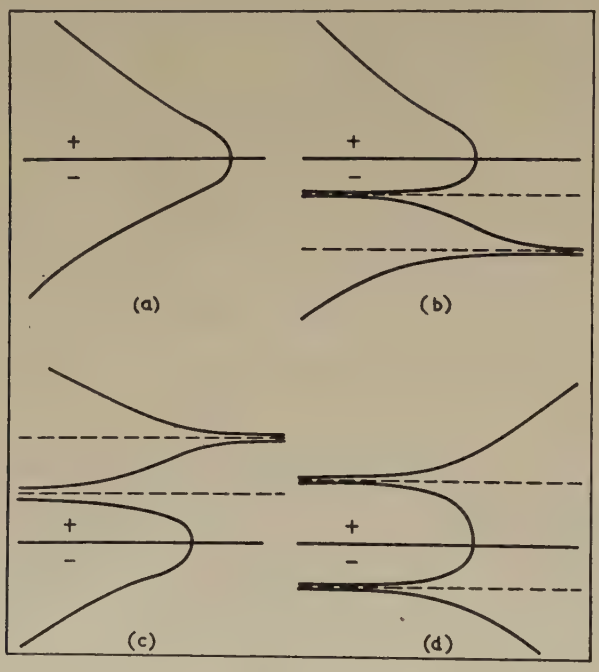


Figure 1. Basic types of solution. h as a function of x. (a) Imaginary asymptotes. $\Delta < 0$ (implies $u_g < c$ and $dh/dx < 0$ if $h > 0$, v_g can be of either sign). (b) Real asymptotes at negative values of h. $v_g^2 > \Delta > 0$, $v_g > 0$ (implies $u_g < c$ and $dh/dx < 0$ if $h > 0$). (c) Real asymptotes at positive values of h. $v_g^2 > \Delta > 0$, $v_g < 0$ (implies $u_g < c$). (d) Real asymptotes, one at a positive value of h and one at a negative value. $\Delta > v_g^2$ (implies $u_g > c$, v_g can be of either sign).

Only solutions for positive values of h are relevant in the present context and we see that the solutions have one, two or three branches in the upper positive half plane and that the slope of h is of constant sign throughout the length of each branch; in meteorological terms each branch is either a warm or a cold front. The various branches of each type represent separate and distinct solutions of the physical problem and must for meteorological purposes be treated separately. We will therefore classify the *branches* of the solutions and discuss their physical characteristics and the conditions under which they occur.

In some of the branches the depth of cold air increases idefinitely either in the positive x-direction (warm front) or in the negative x-direction (cold front), whereas in others the depth of cold air is limited by a horizontal asymptote. For convenience and brevity we shall refer to these two types as deep and shallow respectively and we shall also continue to use the sign of $u_g - c$ as an ana- kata-criterion.

(i) Cold anafront, deep. Formally, this type corresponds to the positive part of either solution (a) or solution (b) and the conditions under which it can occur are either $\Delta < v_g^2$, $v_g > 0$ or $\Delta < 0$, $v_g < 0$. The isobars are curved cyclonically behind the front (apart from the region between the point of inflexion and the surface front) and the wind component parallel to the front becomes increasingly negative. As indicated above, v_g may be of either sign and thus the front may occur either in a definite trough or at a comparatively minor kink in the isobars. Large and positive values of v_g are associated with the most rapid deepening of the cold air and one therefore expects vigorous fronts to occur in definite troughs (see Fig. 2; it should be noted that this and succeeding figures refer to the Northern Hemisphere).

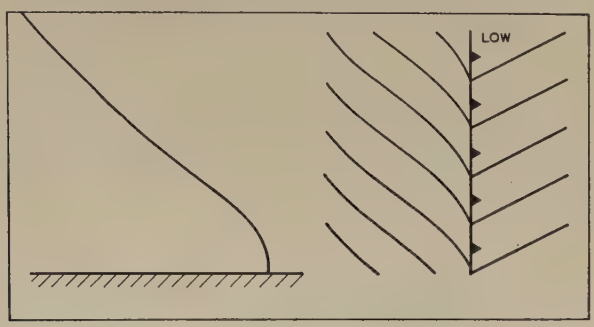


Figure 2. Cold anafront, deep, type (i).

(ii) Cold anafront, shallow. The positive part of the lowest branch of solution (c) is the relevant one in this case, the conditions for which are $v_g^2 > \Delta > 0$ and $v_g < 0$. The depth of cold air increases gradually to a limiting depth given by h_2 (see Table 2 for typical values).

TABLE 2. VALUES OF h_2 IN TENS OF METRES FOR A COLD ANAFRONT, WHERE

$$h_2 = \frac{A_2}{2f}\frac{\eta + [\eta^2 + 4(\xi - 1)]^{\frac{1}{2}}}{\xi - 1}$$

$A/f = 400$ metres $\qquad \eta = v_g/c \qquad \xi = u_g/c$

	ξ							
η	0·8	0·6	0·4	0·2	0·0	− 0·2	− 0·4	− 0·6
− 1·0	110							
− 1·5	59	69						
− 2·0	42	45	49	55	80			
− 2·5	37	34	36	38	40	43	48	
− 3·0	27	28	29	30	31	32	33	35

There is only a transient change in the direction of the isobars and their curvature in the region behind the front is anticyclonic. Such a front can only occur when advancing into a region of lower pressure (i.e., $v_g < 0$). Minor fronts in the cold air stream following a front of type (i), and sea-breeze fronts, if and when such fronts reach a quasi-steady state, appear to be of this type (see Fig. 3). The tendency for sea-breeze air to have a greater depth at lower latitudes is a consequence of the increase in limiting depth with decrease in latitude.

Figure 3. Cold anafront, shallow, type (ii).

(iii) Cold katafront, shallow. This type corresponds to the lowest positive branch of solution type (d), the condition for which is $\Delta > v_g^2$. As in the previous case the depth of cold air increases to a limiting value given by h_2 (see Table 3 for typical values), there is only a transient change in the direction of the isobars and their curvature behind the front is anticyclonic.

TABLE 3. VALUES OF h_2 IN TENS OF METRES FOR A COLD KATAFRONT OR A WARM ANAFRONT

	ξ					
η	1·1	1·2	1·4	1·6	1·8	2·0
− 1·0	38	34	30	28	26	25
− 0·5	67	52	43	38	34	31
0·0	127	90	63	52	45	40
+ 0·5	267	156	93	71	59	52
1·0	438	238	131	95	75	65
1·5	626	325	173	122	96	80
2·0	822	419	218	151	115	97
2·5		516	265	181	139	115

The limiting value is attained more rapidly than in the previous case (as is apparent from Eq. (30) of Section 7), and the reversion of the geostrophic wind to its value preceding the front can take place in as little as two hours. v_g may be of either sign, but the most vigorous fronts will be associated with large, positive values of v_g when the limiting depth is large and is attained rapidly (see Fig. 4).

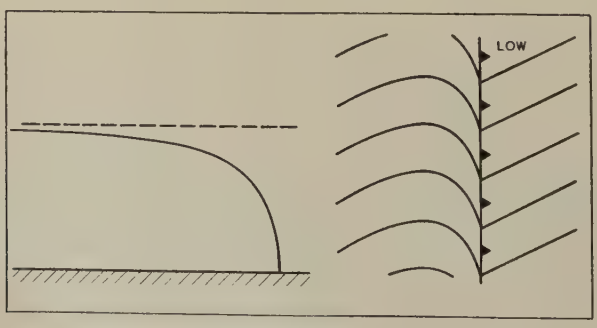


Figure 4. Cold katafront, shallow, type (iii).

(iv) Warm anafront, deep. The upper branch of solution (d) is the relevant one in this case, the condition for which is $\Delta > v_g^2$. The isobars are curved anticyclonically ahead of the front and the frontal slope is steeper for negative values of v_g. The vigorous warm front is therefore not associated with a well-defined surface trough but rather with a change in direction of the isobars. The frontal inversion does not intersect the ground but becomes horizontal, leaving a layer of cold air trailing behind the cold air wedge as depicted in Fig. 5. In Europe such a layer or ' schleppe ' may extend for 100 km behind the wedge (Saucier 1955). The surface warm front corresponds to the position of turbulent breakdown of the schleppe and sharp surface warm fronts can only occur when the schleppe is thin. The depth of the schleppe is given by h_2, values of which smaller than 100 m can only occur at high latitudes (large f), over smooth surfaces (small A), when u_g/c is large and v_g/c is small or negative. This seems the probable explanation why sharp *surface* warm fronts are common only in certain localities at moderate or high latitudes.

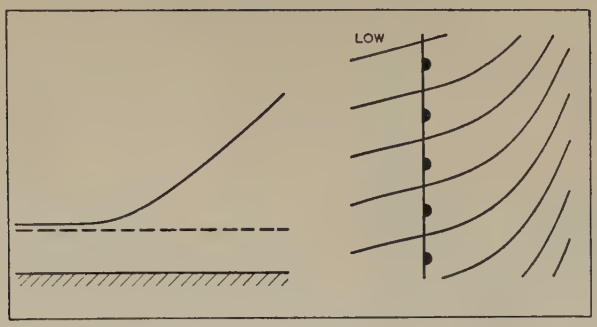


Figure 5. Warm anafront, deep, type (iv).

(v) Warm katafront, shallow. The corresponding branch is the middle one in solution (c), the conditions for which are $v_g^2 > \Delta > 0$, $v_g > 0$. The depth of cold air has a definite upper bound given by h_1 and there is a trailing schleppe of depth h_2 as before (see Fig. 6). This type being a warm katafront, would not be associated with rain or cloud nor would it produce a large wind shift. It does not seem to correspond to any known type of front and presumably if it did occur would easily pass unnoticed.

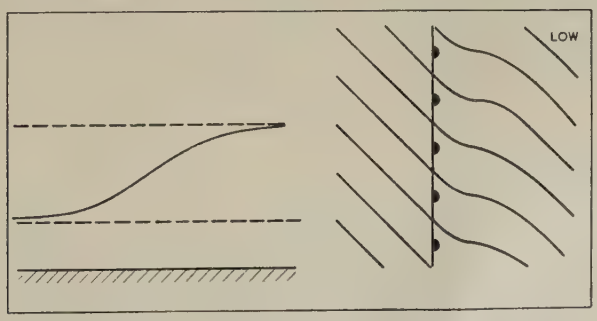


Figure 6. Warm katafront, shallow, type (v).

If the schleppe in cases (iv) and (v) is continuously breaking down then it must be replenished by supply from the main cold-air wedge. Thus the assumption of zero mass flow relative to the system is no longer valid (i.e. $Q \neq 0$) though it may be a good approximation for most purposes. The locations of the warm fronts marked on Figs. 5 and 6 are arbitrary since the position of turbulent breakdown of the schleppe is not known from present considerations.

Only the top branch of solution (c) remains to be discussed. This branch represents a cold anafront in the upper atmosphere. However, the assumption that Q is negligible is completely unjustified in this case and the solution is therefore merely indicative of the possibility of upper cold fronts accompanied by cloud and rain.

Certain types are notably absent from the list of possible solutions, namely : cold katafront, deep; warm anafront, shallow; and warm katafront, deep.

7. The limiting depth

In a number of the solutions classified in Section 6, the cold air tends to a limiting depth given by h_2 (see Eq. (23)). The rapidity with which this depth is attained can be estimated by linearizing Eq. (15) in the following manner. Suppose v equals $v_2 + v'$ where v' is small, then neglecting powers of v', Eq. (15) reduces to :

$$\frac{A_2 v_2 \, dv'}{v' (v_2 - v_1)} = \frac{f^2 (u_g - c) \, dx}{\overset{*}{g}} \quad . \quad . \quad . \quad (28)$$

If v' equals v_0' when $x = x_0$ then the solution is :

$$v' = v_0' \exp [(x - x_0)/\lambda] \quad . \quad . \quad . \quad . \quad (29)$$

where

$$\lambda = h_2 \overset{*}{g}/f \Delta^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

For a typical katafront, type (*iv*), we have $\overset{*}{g} = 10^{-1}$ m sec^{-2}, $f = 10^{-4}$ sec^{-1}, and $\Delta^{\frac{1}{2}} = 10$ m sec^{-1}, whence $\lambda = 100 \, h_2$. Furthermore, h_2 is usually of the order of 1 km for a cold katafront (see Table 3), so that λ is about 100 km.

As a corollary to the above result we conclude that the depth remains approximately equal to the limiting depth, even when this depth varies in response to changes in the upper flow, provided *the variation of the upper flow is gradual in the sense that variations of* h_2 *are small within a distance* λ. This condition can be written :

$$\frac{dh_2}{dx} < 0{\cdot}2 \, h_2/\lambda \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The constant 0·2 has been chosen so that the depth of cold air, having once approximately attained the limiting depth, will remain within about 10 per cent of it.

We can now deduce qualitatively how the depth of cold air will vary behind a katafront situated to the east of the centre of a broad trough. It is assumed that u_g is constant and that v_g varies from negative on the western side of the trough to positive on the eastern side. The limiting depth also increases from west to east, viz :

$$\frac{dh_2}{dx} = \frac{h_2}{\Delta^{\frac{1}{2}}} \frac{dv_g}{dx}$$

and the inequality (31) becomes

$$\frac{dv_g}{dx} < 0{\cdot}2 \, \Delta^{\frac{1}{2}}/\lambda = 0{\cdot}2 f \Delta / h_2 \overset{*}{g}$$

When the same numerical values are taken as before this gives $dv_g/dx < 2 \times 10^{-5}$ sec^{-1}, corresponding to a minimum radius of curvature of about 500 km.

Under these conditions the behaviour of the frontal inversion is indicated in Fig. 7. The cold air attains a maximum depth of 1–2 km at about 100–200 km behind the surface front. At this point the frontal inversion actually intersects the surface representing the limiting depth and thereafter approaches the surface asymptotically from above. The surface pressure pattern is also indicated in Fig. 7. Katafronts exhibiting these general features quite commonly occur in the southern half of Australia. Though they may be isolated, they usually occur as a member of a sequence of cold fronts within a single large trough (Berson, Reid and Troup 1957).

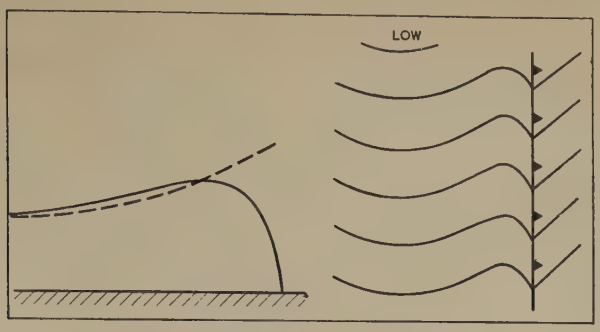


Figure 7. Cold katafront in a broad trough.

8. Comparison with observation

According to the preceding theory, cold katafronts should always be associated with weak surface troughs and small wind shifts, whereas anafronts can be associated with either weak or pronounced surface troughs and may be accompanied by a large wind shift. The cyclonic curvature of the upper flow in the neighbourhood of the front, which is not taken into account in the theory, may occasionally result in a katafront associated with a pronounced surface trough. These conclusions agree with Sansom's (1951) observations. In addition he finds a tendency for anticyclonic curvature of the isobars behind katafronts in broad agreement with the theory. This feature of katafronts is also presumably partly obscured by the general cyclonic curvature of the upper flow.

Sansom also shows that, after an initial sharp rise, the slope of a katafrontal inversion becomes very small, whereas the anafrontal inversion on the average continues to rise. These observations agree with the theoretical prediction that katafronts are always 'shallow.' Sansom, however, deals with the slope derived from wind and temperature observations using Margules's formula. In the present notation this slope can be written $f(v - v_g)/\overset{*}{g}$. According to the theory the actual slope of a katafrontal inversion tends to zero but the slope as calculated from the above formula tends to a constant given by :

$$\frac{f(v_2 - v_g)}{\overset{*}{g}} = \frac{f\Delta^{\frac{1}{2}}}{\overset{*}{g}}$$

which has a typical value of about $+ 1/500$ compared with Sansom's mean measured value of $- 1/300$. Both the values are small and Sansom gives no indication of the standard deviation of his slope values. On the other hand, the slope calculated by this means for a deep anafront at the point of inflexion is given by :

$$\frac{f(v - v_g)}{\overset{*}{g}} = -\frac{f}{\overset{*}{g}}([c(c - u_g)]^{\frac{1}{2}} + v_g)$$

which has a typical value of about $- 1/90$ in good agreement with Sansom's findings. This slope is not very different from the actual slope (see Eq. 27).

We will now consider, as examples, two different types of front. The first of these is the anafront type (i), which traversed parts of South Australia and Victoria on 20 January 1959. The time section at Renmark is depicted in Fig. 8 and the surface pressure distritions at 00 hr and 09 hr (central standard time) are shown in Fig. 9.

The temperature contrast was about 10°C and there was no rain and little or no cloud accompanying the front, a not unusual event during summer in this locality and a fortunate one for comparison with the preceding theory.

The pressure distribution and time sections are in good qualitative agreement with the model, i.e., the front is associated with a definite trough and is a deep anafront. Quantitatively we have the observed values $v_g \simeq 20$ m sec^{-1}, $c = 5{\cdot}7$ m sec^{-1} and u_g is small

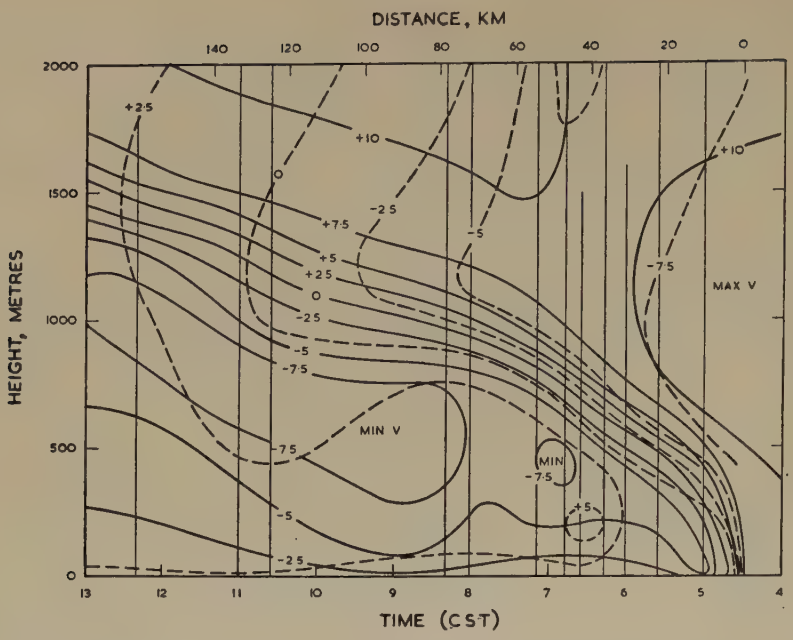


Figure 8. Time section of a cold anafront Renmark 20 Jan. 1959; ——— isopleths of v, – – – – isopleths of $u - c$. The vertical lines indicate double-theodolite pilot-balloon flights.

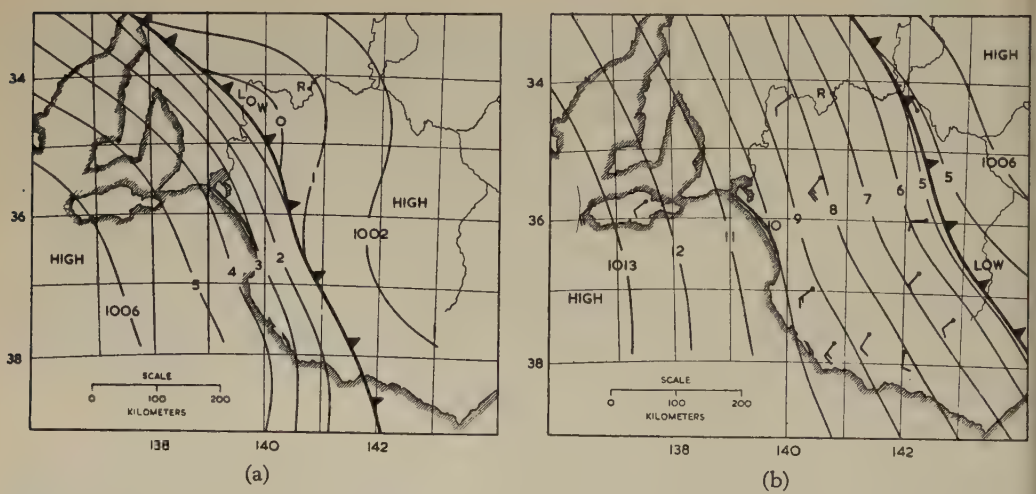


Figure 9. (a) Surface pressures at 00 hr CST 20 Jan. 1959. (b) Surface pressures and winds at 09 hr CST 20 Jan. 1959.

and variable. These values give $\Delta < v_g^2$ and $v_g > 0$ also suggesting an anafront type (i). If we eliminate A from Eqs. (13) and (14) we obtain an expression for the slope of a frontal surface :

$$v\bar{g}\frac{dh}{dx} = f\left[v\left(v - v_g\right) + c\left(c - u_g\right)\right]$$

Estimates of dh/dx using this equation together with observed values of v, lie in the range -7×10^{-3} to -10^{-2}, in excellent agreement with the observed values which lie in the range -7×10^{-3} to -11×10^{-3} (excluding the region within 15 km of the nose).

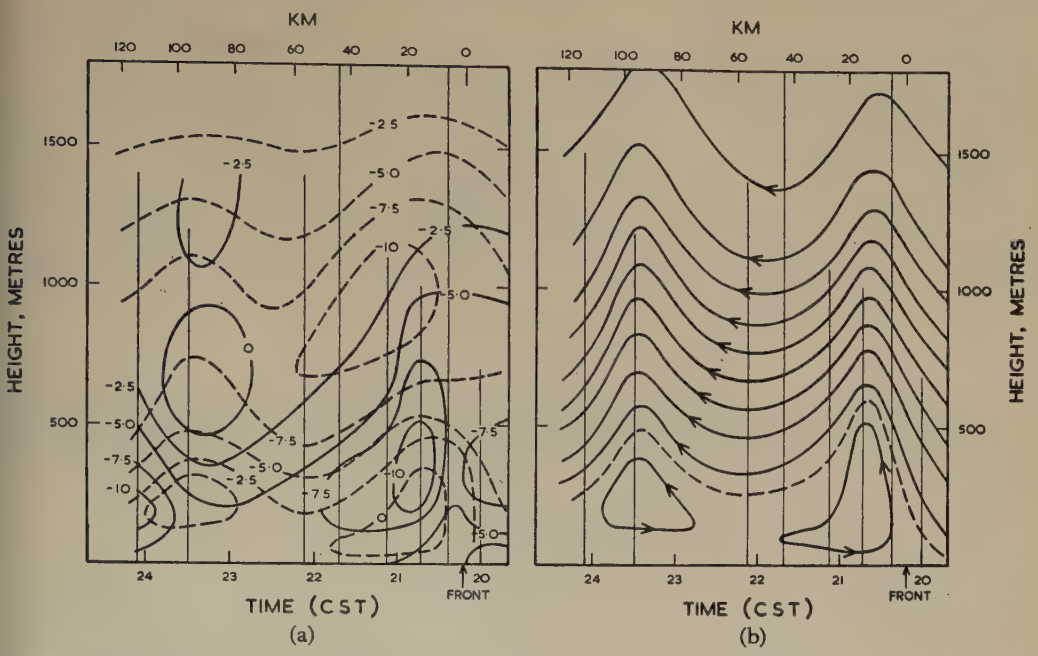


Figure 10. (a) Time section of the ' Renmark surge ' 27 Jan. 1959 ——— isopleths of v, – – – – isopleths of $u - c$. The vertical lines indicate double-theodolite pilot-balloon flights. (b) Streamlines for the ' Renmark surge ' 27 Jan. 1959 derived from the time section by drawing isopleths of ψ, values of ψ having been calculated from the relation $\psi = -\int_0^z u dz$ on the assumption that $\partial v/\partial y$ is zero.

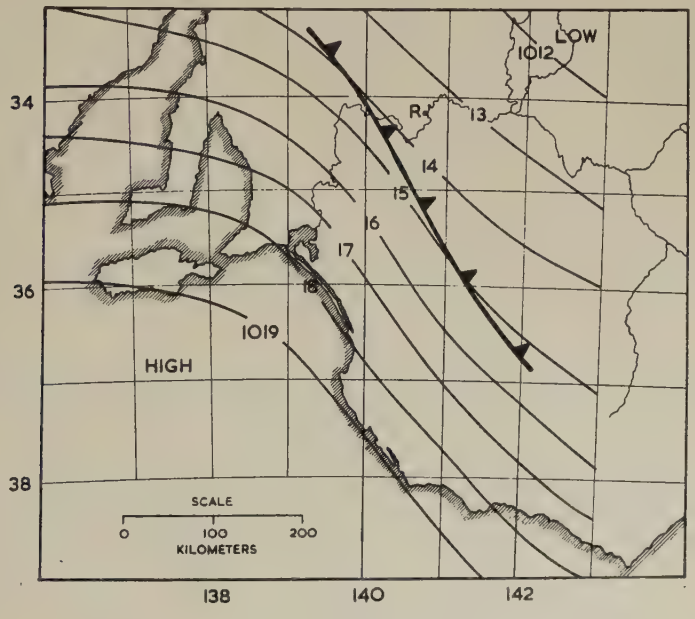


Figure 11. Surface pressures 18 hr CST 27 Jan. 1959.

As the final example we will take the case of the ' Renmark surge ' on 27 Jan. 1959. The Renmark surge is a deeply penetrating sea-breeze front which reaches Renmark (219 km from the sea) in the late evening as a surge of cool air from SSW. This phenomenon at Renmark was first discussed by Reid (1957) and similar phenomena at other places in

Australia (notably Kalgoorlie, Forrest and Canberra) have been considered by Clarke (1956, 1958). A time section and streamline pattern are shown in Fig. 10 and the surface pressures and frontal orientation in Fig. 11).

The qualitative agreement with the theoretical anafront type (ii) is good. Quantitatively we have $v_g \simeq -25$ m sec^{-1}, $c = 8$ m sec^{-1} and $u_g \simeq -3$ m sec^{-1}, thus the fairly stringent conditions for an anafront type (ii) namely $v_g^2 > \Delta > 0$ and $v_g < 0$, are satisfied, The limiting depth calculated from Eq. (23), assuming that $A_2 f$ is 600 m, is about 500 m, in rough agreement with the observed value. We have here referred to the mean depth; the sharp-crested waves which appear on this and on some other cross-sections are beyond the scope of the present theory.

Acknowledgments

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APPENDIX

The intermediate case

When $u_g = c$ we have a special case, intermediate between ana- and kata-types. In these circumstances, from Eq. (14), either $A \equiv 0$ or $v \equiv 0$. If $A \equiv 0$ the usual Margules's equation for the inversion slope is obtained :

$$\frac{dh}{dx} = \frac{f(v - v_g)}{\overset{*}{g}} \quad . \quad . \quad . \quad . \quad (1a)$$

This frictionless case will not be discussed further. If $v \equiv 0$ then from Eq. (13) :

$$\frac{dh}{dx} = -\frac{hf\,v_g + Au_g}{h\overset{*}{g}} \quad . \quad . \quad . \quad . \quad (2a)$$

u and v are both constant, consequently A is also a constant and Eq. (2a) can be integrated to give :

$$\frac{hf\,v_g}{Au_g} - \log\left|1 + \frac{hf\,v_g}{Au_g}\right| = -\frac{f^2 v_g^2 x}{A\overset{*}{g}u_g} \quad . \quad . \quad . \quad (3a)$$

There are three distinct types of solution :

(i) Cold front $v_g > 0$. The depth of cold air continues to increase indefinitely after the frontal passage, the isobars are, however, slightly curved anticyclonically.

(ii) Cold front $v_g < 0$. The depth of cold air never exceeds the value $-Au_g/fv_g$.

(iii) Warm front $v_g < 0$. This is a deep warm front with a trailing layer (schleppe) of depth $-Au_g/fv_g$.

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Similarity theory in the relation between fluxes and gradients in the lower atmosphere

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SUMMARY

Available data on wind and temperature profiles and heat and momentum fluxes in the lower atmosphere have been examined in the light of the similarity theory of Monin and Obukhov (1954) and, except in conditions of very strong lapse where some doubt remains, reasonable agreement is found.

The ' log + linear ' profile form is investigated and it is shown that, in lapse conditions, it is applicable only to a very limited range of stability corresponding approximately with forced convection conditions. The constant, α, which enters into this profile is about 6 and thus considerably larger than the value proposed by Monin and Obukhov. The discrepancy is due to these authors' having applied the profile to a larger range of stability than is justified.

Subject to the reservation expressed for the case of very strong lapse conditions, the similarity scheme has been shown to extend to free convection conditions where a $-\frac{1}{3}$ power form of profile is more appropriate. The relevant constants are estimated.

1. INTRODUCTION

Studies of the important practical problem of relating vertical fluxes (particularly those of heat and momentum) in the lower atmosphere to the appropriate gradients have received considerable impetus from the recent applications of similarity theory. One of the fullest descriptions is given by Monin and Obukhov (1954) and there are subsequent papers by Kazanskii and Monin (1956, 1958). There has been considerable discussion of this work in English, e.g., by Ellison (1957) and Sheppard (1958), but the only comparisons with observation appear to be that by Monin (1953) and the more comprehensive one by Monin and Obukhov (1954).

It is, perhaps, not widely enough recognized (see *Q.J.*, **82**, p. 242) that Monin and Obukhov did not have independent flux measurements. They assumed the theory, deduced fluxes (strictly, certain functions of them) by statistical analysis of the gradient observations and inferred the validity of the theory from the mutual consistency of their results.

The literature contains two sets of observations of fluxes and simultaneous gradients, those reported by Rider (1954) and Swinbank (1955) and it is the purpose of the present paper to examine the similarity theory of the Russian workers in the light of this material. For brevity, these two sets of observations and their authors will be referred to, respectively, as R and S. Monin and Obukhov (1954) will be referred to as M and O.

The basic assumption of the theory is that the effects of deviations from neutral stability are expressed by a scale of length, L, and that mean temperatures and wind speeds, when expressed in dimensionless form, are universal functions of z/L. L is defined by

$$L = -\frac{u_*^3 T_0 C_p \rho}{k g H} \quad . \quad . \quad . \quad . \quad (1)$$

where u_* is the friction velocity, T_0 is the absolute temperature, k is von Kármán's constant (here taken as 0·40), H is the vertical heat flux (considered positive when directed upward) and C_p, ρ and g have their usual meanings. The usual expression of the theory is that

$$\frac{k}{u_*}[u(z_2) - u(z_1)] = f_1\left(\frac{z_2}{L}\right) - f_1\left(\frac{z_1}{L}\right) \quad . \quad . \quad . \quad (2)$$

and

$$-\frac{k\,u_*\,C_p\,\rho}{H}[T(z_2) - T(z_1)] = f_2\left(\frac{z_2}{L}\right) - f_2\left(\frac{z_1}{L}\right) \quad . \quad . \quad (3)$$

and it is usually further assumed that f_2 differs from f_1 only by a constant factor, γ. It is easy to show that, if $f_1/f_2 = \gamma$, then the exchange coefficients for heat and momentum (K_H and K_M respectively) are in the same ratio.

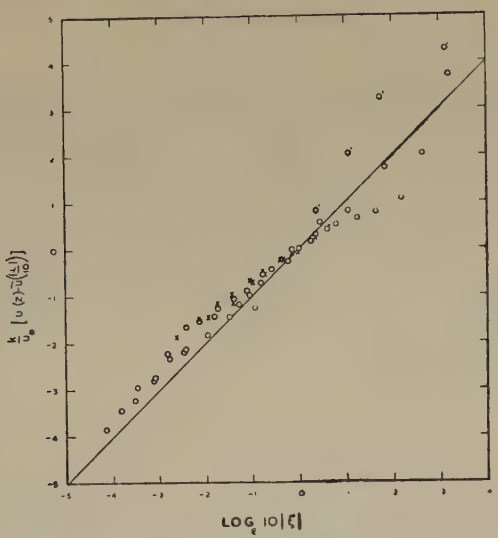

Figure 1. Wind profiles.

x – Rider 1954. o – Swinbank 1955.

2. The observations

Of the numerous results presented in *R* only nine, each representing 20 min observing time, give the full flux and gradient data required for the present purpose. Two of the nine were taken in lapse conditions and, of the remaining seven inversion observations, two had to be omitted from the analysis for a reason which will be explained. Each observation gives, in addition to the fluxes, winds at eight heights and temperatures at six.

The measurements given in S were each of 5 min duration. There were a few in which the measured winds or temperatures did not vary monotonically with height and these have been rejected, as were also five observations in which the measured stresses were negative and two where they were (obviously wrongly) very nearly zero. This selection left 100 runs of which seven could not be used for the same reason as the two R observations referred to. The remaining 93 runs gave fluxes measured at various heights between 1·5 m and 29 m and (with very few exceptions) temperature differences over four height intervals and wind speeds at five heights.

There is a difference of emphasis between the two sets of observations. R concentrated on the very accurate measurement of gradients but his technique of heat-flux measurement seems less reliable than that of S who evaluated it directly from recordings of eddy structure. On the other hand, the gradients reported by S often show more scatter than those of R – an effect to which the shorter observation period may contribute.

TABLE 1. SUMMARY OF OBSERVATIONS USED

(a) Data given by Swinbank (1955)

Group	No. of runs averaged	L (m)	H (mw/cm^{-2})	u_* (cm sec^{-1})	wind speed (cm sec^{-1}) at 0·5 m	1 m	2 m	8 m	32 m	$\Delta\theta$ (°F) 2 m-0·5 m	2 m-1 m	8 m-2 m	30 m-2 m
A	9	+ 45·6	− 1·4	18·6	200	247	293	382	554	+ 0·87	+ 0·44	+ 0·71	+ 1·72
B	9	+ 18·8	− 1·2	13·6	143	185	231	398		+ 1·15	+ 0·62	+ 1·38	+ 3·00
C	8	− 3·39	+ 13·8	17·8	216	270	323	368		− 2·72	− 0·97	− 1·02	− 2·60
D	8	− 12·5	+ 9·18	22·3	243	297	344	410	519	− 2·28	− 0·89	− 1·20	− 1·94
E	7	− 22·2	+ 17·1	31·8	261	317	374	427	537	− 2·43	− 1·00	− 1·53	− 2·08
F	9	− 35·2	+ 17·7	40·5	289	362	438	533	590	− 2·44	− 1·16	− 1·43	− 2·72
G	10	− 60·1	+ 13·8	43·7	333	418	478	590	660	− 1·94	− 1·04	− 1·32	− 2·32
H	10	− 111	+ 11·0	49·1	383	514	562	690	819	− 1·56	− 0·72	− 1·32	− 2·18
I	9	− 168	+ 8·6	51·4	318	446	532	662	782	− 0·94	− 0·57	− 0·83	− 1·26
J	8	− 225	+ 4·9	46·2	368	443	522	654	805	− 0·84	− 0·30	− 0·58	− 1·11
K	6	− 320	+ 5·6	51·2	348	461	542	702	839	− 0·72	− 0·43	− 0·58	− 1·08

(b) Data given by Rider (1954)

Observation No.	L (m)	H (mw/cm^{-2})	u_* (cm sec^{-1})	wind speed (cm sec^{-1}) at 2 m	1·5 m	1 m	0·75 m	0·5 m	0·375 m	0·25 m	0·15 m	Temperature (°F) at 2 m	1·5 m	1 m	0·5 m	0·375 m	0·25 m
29	+ 9·25	− 3·6	15·7	238	216	194	182	164	154	140	121	46·3	45·98	45·64	45·18	45·02	44·77
30	+ 30·2	− 6·7	28·6	393	368	333	325	292	279	260	225	46·0	45·84	45·63	45·26	45·13	44·93
31	− 20·8	+ 10·5	29·4	374	360	341	327	307	292	267	237	49·1	49·25	49·48	50·01	50·26	50·64
33	− 10·2	+ 4·1	16·9	215	205	194	186	173	166	148	134	51·0	51·03	51·15	51·37	51·62	51·80
44	+ 44·6	− 8·9	35·8	577	549	506	490	442	427	390	352	47·3	47·18	47·11	46·71	46·60	46·40
48	+ 5·08	− 3·4	12·6	264	250	232	214	197	187	168	146	40·1	39·94	39·66	39·18	39·00	38·69
50	+ 18·3	− 3·5	19·5	307	292	272	253	232	221	197	172	39·5	39·39	39·23	38·78	38·64	38·37

The form of the calculations to be undertaken makes it necessary to interpolate winds and temperatures to a certain reference height (expressed as a function of $|L|$) and therefore, to obtain more reliable interpolated values, Swinbank's winds and temperatures were averaged in groups of about eight while Rider's were taken individually. (It is because of the unattainability of the desired reference height within the observed profiles or by reasonable extrapolation from them that certain observations were omitted as noted above).

Another minor point of difference is that S tabulates potential temperature differences and R the Fahrenheit temperatures. In view of the small heights used by the latter, the distinction has been ignored and the analysis carried through in terms of potential temperature (θ). The observations are summarized in Table 1. In the case of S, a value of L was worked out for each individual run; they were then arranged into groups by ranges of L and mean H, u_*, winds and temperatures calculated for each group.

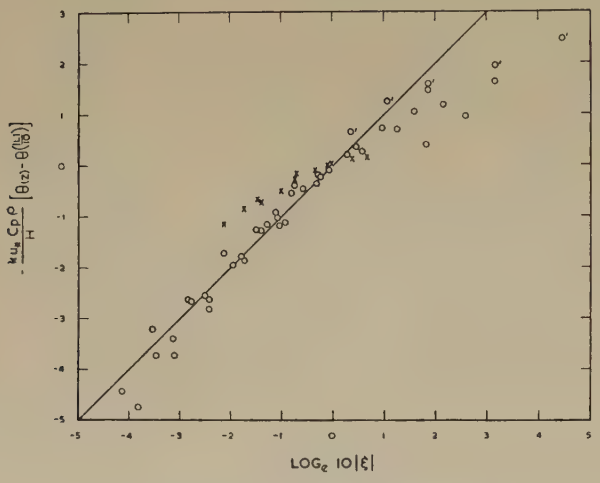

Figure 2. Temperature profiles.

x – Rider 1954. o – Swinbank 1955.

3. Preliminary application of similarity principles

To reduce wind and temperature differences with varying values of z and L to a form where it is possible to examine the validity of Eqs. (2) and (3), it is clearly necessary to choose a reference height, z_1, related to the value of L. Only then will all points lie on a common curve if these suggested forms are correct. M and O used $\frac{1}{2}|L|$ as their reference height, but to have done so here would have meant the abandonment of a good part of the observations where profile measurements did not extend up to such a height. The reference height chosen was $0{\cdot}1\,|L|$ so that Eqs. (2) and (3) can be written as :

$$\frac{k}{u_*}[u(z) - u(0{\cdot}1\,|L|)] = f(\xi) - f\,0{\cdot}1\left(\frac{|L|}{L}\right) \quad . \quad . \quad (4)$$

and

$$-\gamma\frac{k\,u_*\,C_p\,\rho}{H}[\theta(z) - \theta(0{\cdot}1\,|L|)] = f(\xi) - f\,0{\cdot}1\left(\frac{|L|}{L}\right) \quad . \quad . \quad (5)$$

where $\xi = z/L$ and $\gamma = K_H/K_M$.

The observations have been reduced to this form and are shown graphically in Figs. 1 and 2, in the latter of which γ is put equal to unity. In these figures the left-hand sides of Eqs. (4) and (5) are displayed against $\log_e 10\,|\xi|$. Lapse conditions (L negative) only are included since lapse and inversion observations lie on different branches of the curve.

In the case of the wind profiles, there is little doubt that most of the observations fall on a common curve within the scatter to be expected in this type of analysis. Of the divergent points, those marked with a prime, comprise Group C, made in the strongest lapse conditions. In the temperature profiles, Group C (similarly marked) is again divergent, though not so strongly. The R and S observations appear to show internal agreement but to differ systematically from one another. These discrepancies will be commented on further. The wind and temperature curves, while broadly similar in form, are not identical, which implies that the assumption of a constant γ equal to unity is incorrect.

The usual expression for the logarithmic wind profile appropriate to adiabatic conditions can be put in the form

$$\partial u/\partial z = u_*/kz \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{k}{u_*}\frac{\partial u}{\partial (\log |\xi|)} = 1 \quad . \quad . \quad . \quad . \quad (7)$$

Reference to Fig. 1 shows a large deviation from this slope at values of $|\xi|$ considerably less than 0·1 so that the thickness of the layer of ' dynamical ' turbulence, where thermal influences are unimportant, is probably quite a small fraction of $|L|$.

4. The ' log + linear ' profile

Since the wind profile is logarithmic under conditions sufficiently near neutral, it was suggested (apparently first by Monin 1953) that $f(\xi)$ might be represented over a wider range of stabilities by

$$f(\xi) = \log_e |\xi| + \alpha\, \xi^* \quad . \quad . \quad . \quad . \quad (8)$$

If this is so, then Eq. (4) can be put in the form

$$\frac{k}{u_*}[u(z) - u(0{\cdot}1\,|L|)] = \log_e 10\,|\xi| + \alpha\left(\xi - 0{\cdot}1\,\frac{|L|}{L}\right) \quad . \quad . \quad (9)$$

and α can be found by ordinary regression methods.

When this is done with the whole of the S data, we obtain $\alpha = 0{\cdot}1$. The precision of this determination is very low as can be realized from Fig. 1, considering that α is determined by the deviations of the plotted points from the line of unit slope through the origin – in fact, the standard error is 0·2. M and O report a value of 0·6 for α and claim an accuracy of 10 per cent. This claim is hard to understand when one contemplates their Fig. 4, or better, the transformation of it published by Priestley (1959) but in any case, the significance of the difference between the two determinations is open to doubt.

A different picture emerges from R where, taking all observations together, α appears to be 2·9†. This prompts a closer examination of the observations. The mean value of $|L|$ reported by M and O is 21 m with heights of observation ranging up to 15 m. For S, the corresponding figures are 92 m and 32 m and for R, they are 20 m and 2 m. Thus, while the first two sets of data cover comparable ranges of $|\xi|$, the third is somewhat apart in this respect. The distribution of observations between lapse and inversion also differs from one set to another.

It is reasonable to expect that the range of ξ might affect the value of α calculated as above because Eq. (8), if valid at all, can only be so for small enough $|\xi|$. Just what range of $|\xi|$ this implies is a matter for experiment but, at least in lapse conditions, it is unlikely that there exists any other criterion having as great a physical significance as the boundary between forced and free convection which Priestley (1955) has shown to exist at a Richardson number (Ri) of about $-0{\cdot}03$.

* α denotes the constant for which Monin wrote β. This, as noted by Sheppard (1958), allows β to be reserved for another, earlier application.

† Businger's (1955) analysis of R is consistent with a value of 2 for α and not 0·8 as stated by Ellison (1957).

It is easy to show that

$$Ri = \frac{1}{\gamma\, \partial f/\partial \xi}$$

and thus, if Eq. (8) is valid and $\gamma = 1$, it follows for small enough ξ that

$$Ri = \xi \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (10)$$

This relationship can be checked directly from the observations. R gives values of Ri at three heights (1·5, 0·75 and 0·375 m) and S at one (1·5 m). The corresponding ξ have been calculated and the results (excluding a few at large $|Ri|$ and $|\xi|$ which would have forced a very compressed scale on the graph) are plotted in Fig. 3, which clearly shows some systematic divergences from Eq. (10). However, the critical Richardson number separating forced from free convection is not very closely determined and the value of γ is uncertain so we shall, therefore, first analyse the observations for which $-0{\cdot}03 \leqslant \xi < 0$ and refer to these as representing forced convection conditions.

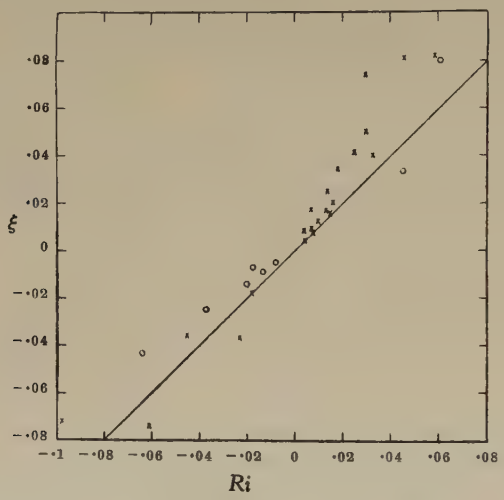

Figure 3. Relationship between ξ and Ri.

x – Rider 1954. o – Swinbank 1955.

5. Forced convection

If we accept that α should be calculated from observations made in forced convection conditions, it is clearly necessary that the reference level (previously taken as $0{\cdot}1\,|L|$) should be low enough to be included in the same regime. For this reason, all wind speeds at $|\xi| \leqslant 0{\cdot}03$ have been re-analysed, using $u\,(0{\cdot}03\,|L|)$ as the reference wind. The results are :

S (18 wind speed measurements) $\alpha = 6{\cdot}1 \pm 1{\cdot}5$

R (6 wind speed measurements) $\alpha = 11{\cdot}7 \pm 1{\cdot}5$

These values may be compared with the range 2·5-5 suggested by Panofsky, Blackadar and McVehil (unpublished) as probably correct for α when $|\xi|$ is small.

Care should be exercised in interpreting the standard errors given. Though they truly represent the scatter of the calculated α in each case, it must be remembered that α does not come from the regression of one independently observed variable on another but from variables into which some spurious correlation has been introduced by the inclusion of L in both.

A test of the homogeneity of the 'forced convection' observations was made by calculating α from those S observations (11 in all) for which $|\xi| < 0{\cdot}01$. The value obtained was $6{\cdot}2 \pm 1{\cdot}9$, thus amply confirming the constancy of α in these conditions. Unfortunately R has too few forced convection observations for any subdivision of them to be useful.

To show more clearly the effect of calculating α from observations at large $|\xi|$, values have been calculated from the remaining lapse observations at various ranges of $|\xi|$. They show

(1)	S	$0{\cdot}03 < \|\xi\| \leqslant 0{\cdot}1$	$\alpha = 1{\cdot}1$
		$0{\cdot}1 < \|\xi\| \leqslant 0{\cdot}5$	$\alpha = 0{\cdot}4$
		$0{\cdot}5 < \|\xi\|$	$\alpha = -0{\cdot}1$
(2)	R	$0{\cdot}03 < \|\xi\|$	$\alpha = 3{\cdot}7$

Particularly in the S data, the systematic variation of the calculated α with $|\xi|$ confirms that even in the range 0·03-0·1, $|\xi|$ is too large for the application of Eq. (8).

A closely similar variation of the calculated α with stability is demonstrable in the M and O data though it was not, apparently, recognized by them. They use a stability parameter denoted by (the Russian form of) the letter B.

$$B = \frac{g}{T_0} \frac{T_2 - T_{0{\cdot}5}}{u_1^2}$$ where the subscripts (other than 0) represent heights in metres.

Having, by means of a regression method which they describe, determined α/L for each mean profile, they proceed to obtain an overall value for α (equal to 0·62) by least squares methods from their Eq. (48), here written in the present notation

$$B = \frac{1}{\alpha} \Phi \left(\frac{\alpha}{L} \right) \qquad . \quad . \quad . \quad . \quad (11)$$

where Φ is a determinate function of α/L and the heights used in defining B. However, since they tabulate B and α/L for each profile, it is clearly possible also to calculate individual values of α from Eq. (11). This has been done for their lapse results (B positive) omitting

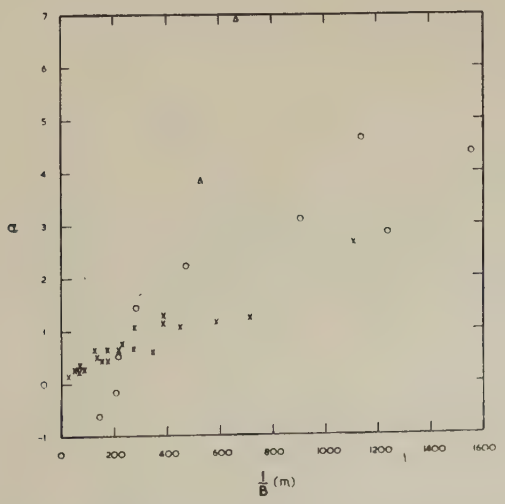

Figure 4. Variations of α with stability.

x – Monin and Obukhov 1954. △ – Rider 1954. o – Swinbank 1955.

those where α/L was also reported as positive and two other profiles where there were obvious misprints of essential figures. The results are given in Fig. 4 and the systematic variation with stability of the calculated α is clearly shown. For comparison, the S and R data have been reduced to this form and are also plotted in Fig. 4.

Since a value of α can be obtained for each mean M and O profile, better values of L than those tabulated by the authors can be calculated. Even though most of their results are outside the forced convection range, this procedure is approximately justified because α is not considered constant but adjusted separately to each profile, the ' log + linear ' expression being used simply as an interpolation formula. Values of L so calculated were used to select the forced convection observations from the M and O tables and to obtain an appropriate value of α. The comparison with values given above is a little complicated by the fact that the calculated value of α is rather sensitive to that allotted to k, M and O having put $k = 0{\cdot}43$ as against the value 0·40 used in the present work. Using $k = 0{\cdot}43$, the value of α obtained is 2·7 while, if the data are adjusted so that $k = 0{\cdot}40$, α is calculated as $6{\cdot}0 \pm 0{\cdot}9$. It is clear from the results and Fig. 4 that the M and O observations are in better agreement with S than with R.

Comparison of Figs. 1 and 2 suggested that the value of γ was not equal to unity. It is possible to evaluate γ from the temperature profiles in forced convection conditions by using the value of α previously determined from wind speed profiles. For the case of S, least squares methods are used to solve

$$-\gamma \frac{k u_* C_p \rho}{H} [\theta(z) - \theta(0{\cdot}03\,|L|)] = \log_e 33{\cdot}3\,|\xi| + 6{\cdot}1\left(\xi - 0{\cdot}03\,\frac{|L|}{L}\right)$$

and a similar equation is solved for the R data. The results obtained are for S, $\gamma = 0{\cdot}76$; for R, $\gamma = 1{\cdot}2$. Fig. 5 displays the forced convection observations from S in comparison with the mean profile derived from them

$$\left.\begin{aligned} &\frac{k}{u_*}[u(z) - u(0{\cdot}03\,|L|)] \\ -\,0{\cdot}76\,&\frac{k u_* C_p \rho}{H}[\theta(z) - \theta(0{\cdot}03\,|L|)] \end{aligned}\right\} = \log_e 33{\cdot}3\,|\xi| + 6{\cdot}1\left(\xi - 0{\cdot}03\,\frac{|L|}{L}\right) \quad . \quad (12)$$

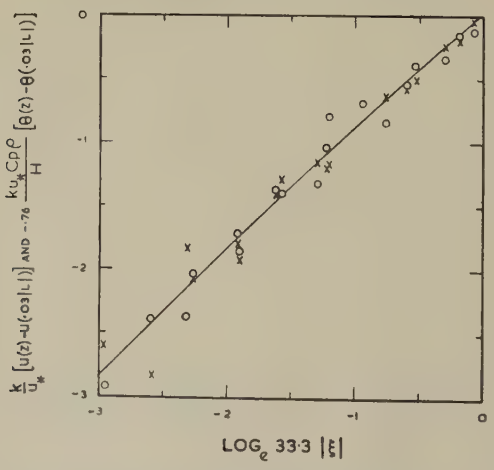

Figure 5. Forced convection wind and temperature profiles.

o – Wind. x – Temperature. — Mean profile.

6. Free convection

The form of the temperature profile in free convection was defined by Priestley (1954) by dimensional analysis and his result was later (1955) expressed in the convenient form

$$\overset{*}{H} = \frac{H}{\rho\, C_p \left(\frac{g}{\theta}\right)^{\frac{1}{2}} \left|\frac{\partial\theta}{\partial z}\right|^{\frac{3}{2}} z^2} = \text{const.}$$

Kazanskii and Monin (1958) use this result but express it in the form

$$-\gamma \frac{k\, u_*\, C_p\, \rho}{H} [\theta(z_2) - \theta(z_1)] = c \left[\left(\frac{z_2}{L}\right)^{-\frac{1}{3}} - \left(\frac{z_1}{L}\right)^{-\frac{1}{3}}\right] \quad . \quad . \quad (13)$$

The latter presentation of the free convection law is less realistic than the former because it apparently makes the profile depend on u_* and K_M (which is included in γ). However, the right-hand side of Eq. (13) contains u_* (in L) in such a way that it can be eliminated from the equation and c must similarly contain K_M in such a way that c/γ does not. This is evident since the latter is an absolute constant related to $\overset{*}{H}$ by

$$\overset{*}{H} = \left(\frac{3\gamma}{c}\right)^{\frac{2}{3}} k^2$$

The available temperature profile observations for $|\xi| > 0{\cdot}03$ in lapse conditions have been examined by determining the regression of $-\frac{k\, u_*\, C_p\, \rho}{H} [\theta(z) - \theta(0{\cdot}1\,|L|)]$ on $\xi^{-\frac{1}{3}}$. Since group C of S has been seen in Figs. 1, 2 and 4 to diverge from the other observations it has been omitted from the calculation and will be discussed separately later. The relationship obtained between these two variables should be of the form

$$-\frac{k\, u_*\, C_p\, \rho}{H} [\theta(z) - \theta(0{\cdot}01\,|L|)] = \frac{c}{\gamma} [\xi^{-\frac{1}{3}} + 0{\cdot}1^{-\frac{1}{3}}] \quad . \quad . \quad (14)$$

No attempt was made in the calculation to restrict the value of the constant term in the regression equation. Nevertheless the two sets of observations give the left-hand side of Eq. (14) equal to

R $(0{\cdot}51 \pm 0{\cdot}04)\ (\xi^{-\frac{1}{3}} + 2{\cdot}14)$

S $(1{\cdot}06 \pm 0{\cdot}05)\ (\xi^{-\frac{1}{3}} + 2{\cdot}15)$

and it is to be noted that $0{\cdot}1^{-\frac{1}{3}} = 2{\cdot}16$. These values of c/γ correspond to $\overset{*}{H}$ of 2·28 (R) and 0·76 (S).

The dimensional analysis for the form of the temperature profile in free convection relies on the physically realistic assumption that $\partial\theta/\partial z$ is independent of u_* in these conditions. No such guiding principle is available for the wind profile and any extension of similarity of the two types of profile to free convection is a matter to be tested. If a form

$$\frac{k}{u_*} [u(z) - u(0{\cdot}1\,|L|)] = c\, [\xi^{-\frac{1}{3}} + \text{const}]$$

is assumed, an analysis similar to that for temperature can be carried out. This was done with the result

R $c = 0{\cdot}85 \pm 0{\cdot}03$

S $c = 1{\cdot}43 \pm 0{\cdot}14$

The additive constants are satisfactorily close to $0{\cdot}1^{-\frac{1}{3}}$, being 2·19 and 2·20 respectively. Values of γ can now be obtained and are

$$\text{R} \quad \gamma = 1{\cdot}67$$

$$\text{S} \quad \gamma = 1{\cdot}35$$

Fig. 6 shows one set of observations (S) in comparison with the calculated curve of ordinates equal to $1{\cdot}43\ (\xi^{-\frac{1}{3}} + 0{\cdot}1^{-\frac{1}{3}})$. The results from Group C, though not included in the regression calculation, are shown marked with a prime.

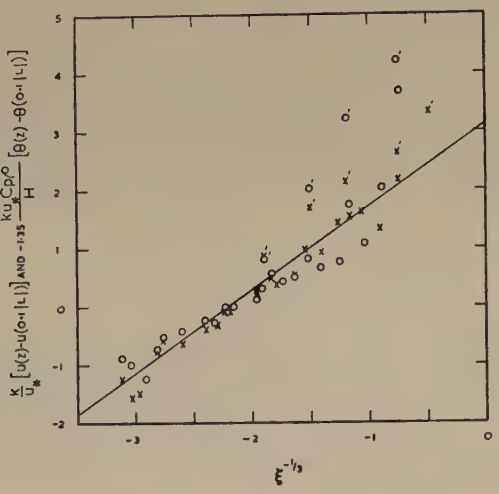

Figure 6. Free convection wind and temperature profiles (Swinbank 1955).

o – Wind. x – Temperature. — Mean profile.

The M and O observations for $|\xi| > 0{\cdot}03$ have been similarly analysed using the improved values of L obtained as described above. The results are shown in Fig. 7 and the regression equation for these points is

$$\frac{k}{u_*}[u(z) - u(0{\cdot}1\,|L|)] = (1{\cdot}31 \pm 0{\cdot}03)\ (\xi^{-\frac{1}{3}} + 2{\cdot}22)$$

which shows no significant difference from that applicable to S. The small difference between the values of k used is unimportant here as no part of the calculation is unduly sensitive to it.

Fig. 7 shows an obvious departure from a linear relationship at large $|\xi|$ similar to, but not so marked as, that exhibited by Group C in Fig. 6. Deacon (1955) has discussed the tendency of the S observations to underestimate shearing stresses at heights of about 20-30 m in strong lapse conditions and from that point of view, the Group C observations are under some suspicion. However, no plausible adjustment of measured stresses would completely reconcile this group with the other observations. This feature is also discussed by Webb in a treatment (in preparation) in which wind profiles are examined by a method previously applied to temperature profiles (Webb 1958). However, before this condition is approached, Fig. 7 shows the very good agreement of the M and O data with a $-\frac{1}{3}$ power law for values of $|\xi|$ from about 0·03 onwards.

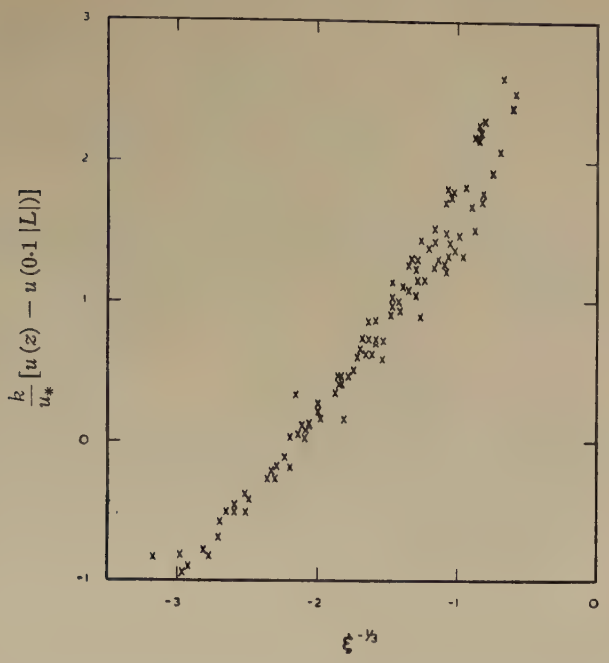

Figure 7. Free convection wind profiles (Monin and Obukhov 1954).

7. Inversion conditions

It is possible that Eq. 8 is just as much restricted to small $|\xi|$ in inversion as it is in lapse conditions but it is not necessarily so because a linear profile is the limiting form as turbulence decreases with increasing stability. In the present work, no subdivision of inversion observations has been made, partly because of the smaller available range of $|\xi|$, compared with the lapse data, and partly because of the absence of a criterion of such physical significance as the difference between free and forced convection. The inversion data were analysed in a manner similar to that described above for forced convection, with the results

$$\text{R} \quad \alpha = 2{\cdot}4 \pm 0{\cdot}3, \quad \gamma = 1{\cdot}84$$

$$\text{S} \quad \alpha = 5{\cdot}8 \pm 1{\cdot}3, \quad \gamma = 0{\cdot}99$$

but it must be allowed that these values of α may be contaminated by the inclusion of observations at too large $|\xi|$.

8. Conclusions

The numerical results obtained above may be summarized as in Table 2.

TABLE 2. Summary of results

		R	S	M and O
Forced convection	α	$11{\cdot}7 \pm 1{\cdot}5$	$6{\cdot}1 \pm 1{\cdot}5$	$6{\cdot}0 \pm 0{\cdot}9$
	γ	1·2	0·76	
Free convection	c	$0{\cdot}85 \pm 0{\cdot}03$	$1{\cdot}43 \pm 0{\cdot}14$	$1{\cdot}31 \pm 0{\cdot}03$
	γ	1·67	1·35	
Inversion	α	$2{\cdot}4 \pm 0{\cdot}3$	$5{\cdot}8 \pm 1{\cdot}3$	
	γ	1·84	0·99	

There are some obvious discrepancies between results obtained from R and from S. As far as γ is concerned, at least, values of K_H and K_M given by these two authors make it clear that the discrepancies are inherent in the observations and are not merely consequent upon the present method of analysis. Wherever a comparison is possible the results from the M and O observations agree very much better with those from S than with those from R.

Moreover, it can be shown* that the forced convection value of α obtained above from R ($\alpha = 11{\cdot}7$) is inconsistent with this author's own analysis of his wind-profile data. R gives (his Fig. 8) a graph of $\tau/\rho z^2 \left(\frac{\partial u}{\partial z}\right)^2$ against Richardson number (τ is the shearing stress, equal to ρu_*^2). Making use of the fact that $Ri = \xi$ when both are small, this graph indicates that

$$\frac{\partial}{\partial \xi}\left[u_*^2/\xi^2 \left(\frac{\partial u}{\partial \xi}\right)^2\right]_{\xi=0} = -1{\cdot}8$$

From the definition of the ' log + linear ' profile, it follows that

$$\frac{\partial}{\partial \xi}\left[u_*^2/\xi^2 \left(\frac{\partial u}{\partial \xi}\right)^2\right] = -2k^2\alpha(1+\alpha\xi)^{-\frac{1}{2}}$$

so that, near $\xi = 0$, $\alpha = 5{\cdot}5$ which is in satisfactory agreement with the value $\alpha = 6{\cdot}1$ obtained from S.

This derivation of α from R uses, of course, all the wind speed data obtained by him in near-neutral conditions, while it must be pointed out again that all the lapse results for R appearing in Table 2 are derived from only two independent observation periods. The method of heat flux measurement used by Rider is, *a priori*, open to criticism which is strengthened by the evidence of Fig. 2 and the comparisons of α given above, so that there are strong reasons for preferring the results obtained from S.

Except in strong lapse conditions, where some doubt remains, the similarity scheme of Monin and Obukhov has been shown to apply. The ' log + linear ' profile has been confirmed and its limit of applicability, on the lapse side, has been shown to be close to $|\xi| = 0{\cdot}03$, corresponding approximately with forced convection conditions. The value of the constant, α, entering into this profile is about 6, that is, it is of the order of ten times bigger than was suggested by Monin and Obukhov, the error in whose value was due to the application of the profile to too wide a range of $|\xi|$.

* This approach is due to Mr. E. L. Deacon.

9. Acknowledgments

I am indebted to Mr. E. L. Deacon, Dr. C. H. B. Priestley and Mr. E. K. Webb for much helpful discussion and criticism.

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Measurements of space charge in the lower atmosphere using double field mills

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SUMMARY

Two double field mills (Smiddy and Chalmers 1958) have been used to study space charge in the lowest 5 m of the atmosphere.

Results in fair weather show small values, often negative, which can be explained in terms of radioactivity of the ground. Comparison with results from filtration apparatus suggest that the latter may be in error when small ions are present in large numbers.

Results in light or moderate rain show small space charges, but in heavy rain, negative space charges of the order of $-1{,}000\ \mu\mu c\ m^{-3}$ are found, probably formed by splashing. In snowfall, there are charges of both signs of about $\pm 500\ \mu\mu c\ m^{-3}$.

1. INTRODUCTION

The space charge in the lowest few metres of the atmosphere is of importance both in fair and disturbed weather. In fair weather, the interest lies in connexion with the electrode effect (see Section 3) and in disturbed weather with charges produced by the splashing of rain.

Particularly for discussion of the electrode effect, it is essential that the measurements should involve as little disturbance as possible of the natural conditions. Most of the methods previously used for measuring space charges have inevitably required some disturbance and are therefore not satisfactory. What is needed is some apparatus which leaves the lines of force, etc., as close as possible to their condition in the absence of the apparatus.

The present work can be considered to be preliminary observations which not only establish the method but also give results of some interest.

2. RELATION BETWEEN SPACE CHARGE AND CONDUCTIVITY

Apart from precipitation currents, the electric currents that exist in the lowest regions of the atmosphere can be classified as convection currents and conduction currents. Convection currents can carry charge if there are space charges, but if these currents are averaged over space or time so as to eliminate any excess of upward or downward motion, then any electric current must depend upon the existence of a gradient of space charge. The actual values of space charges and their gradients are so small that it is unlikely that the convection currents are appreciable in the lowest layers of the atmosphere in fair weather.

Neglecting convection, if we consider a region of the atmosphere in which there are quasi-static conditions, i.e., in which instantaneous pictures of the distribution of charge would be identical, although the actual charges are moving under the influence of the potential gradient, then the vertical conduction current density must be the same at all levels.

Putting
$$i = \lambda F,$$
where i is the conduction current density, λ the specific conductivity and F the potential gradient.
$$F = i/\lambda = i\,r$$
where $r = 1/\lambda$ is the specific resistivity.

Thus $$\frac{dF}{dx} = i\frac{dr}{dx}$$

From Poisson's law, $\rho = -\epsilon_0 \frac{dF}{dx}$ where ρ is the space charge density and ϵ_0 is the permittivity of space

$$\rho = -\epsilon_0 \frac{dF}{dx} = -\epsilon_0 i \frac{dr}{dx}$$

In a region between heights x_1 and x_2, where the resistivities and conductivities are r_1, λ_1 and r_2, λ_2 respectively, there will be a charge per unit area

$$q = -\epsilon_0 i (r_2 - r_1)$$

$$= \epsilon_0 i \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

It is to be noted that the conductivity in any region of the atmosphere depends not only on the production and destruction of ions in the region concerned, but also on the movement of ions into and out of the region, by conduction and convection.

3. The electrode effect

In the normal fair-weather positive potential gradient in the atmosphere, ions move in the electric field, positive downwards and negative upwards; there can be no negative ions close to the ground unless they are produced at the earth's surface. Therefore, we should expect a decrease of conductivity, and so, from the results of section 2, a positive space charge and a higher potential gradient. This is known as the 'electrode effect' and calculations have shown that it would be expected that the potential gradient would be about 30 per cent higher at the ground than at 1 m, which would involve an average space charge density of about + 300 $\mu\mu$c m^{-3} over the first metre. Actual measurements show that the space charges and potential-gradient changes are much smaller than these values, and various attempts have been made to resolve this discrepancy.

The most promising suggestion is that made originally by Hogg (1939), and worked out in more detail by Chalmers (1946), that the conductivity close to the ground is increased by the effect of α – and β – radiation from radioactive substances in and very close to the earth's surface. The conductivity due to such sources would decrease with height and would, on its own, give a negative space charge; it appears that the positive space charge from the electrode effect and the negative space charge from radioactive effects balance approximately, to give an approximately uniform potential gradient with small space charges.

4. Methods of measuring space charge

A thorough review of the methods by which space charge can be measured has been given recently by Vonnegut and Moore (1958). From this, it is quite clear that most of the methods involve setting up apparatus above the earth's surface, but at earth potential, thus unavoidably disturbing the lines of force. In addition, those methods which involve air being drawn into a measuring apparatus are disturbing the natural condition of the air and charges induced on the apparatus will affect the charges in the air sampled.

The only type of method which can avoid disturbances is the one in which the space charge is obtained by Poisson's formula from the measurement of potentials or potential gradients, but previous methods of this type have created their own disturbances by putting particles or ions into the air. The ideal is to use some form of apparatus for the measurement of potential or potential gradients which does not disturb natural conditions, and this is approached by the double field mill (Smiddy and Chalmers 1958).

5. Apparatus

The double field mill has been described (Smiddy and Chalmers 1958) in detail; suffice it to say here that it consists of two conventional field mills mounted on the same vertical shaft and pointing one upwards and one downwards. If the mounting of the double mill is at the potential of its surroundings, the two mills will give equal and opposite outputs; the outputs are added electronically and the resultant actuates a servo-mechanism which adjusts the potential until the system is brought to the potential of its surroundings; at the same time, the output of one mill gives a value for the potential gradient. Two such double field mills were mounted on girders, one with its ends at 1 m and the mill itself at 85 cm above the ground, and the other at either 3 m and 2·85 m, or 5 m and 4·84 m. The girders were supported by wooden posts and it was considered that the wood would be neither too conducting to be at a potential appreciably different from its surrounding, nor too insulating so as to pick up static charges.

During a number of the recording periods, records were also available of the potential gradient at the ground, given by the agrimeter (Chalmers 1953), situated 14 m horizontally from the double field mill, and of the rain current and rate of rainfall at 13 m away.

In addition, a filtration apparatus, of the type used by Obolensky (1925) was set up at 5 m, with its inlet at a height of 65 cm.

6. Calculation of space charge

It was found most convenient to use the values obtained at intervals of 1 min, rather than to attempt to obtain average values.

If F_L, F_U are the values of the potential gradient found at the two mills, then the mean space charge over the height range h is given by

$$\rho = \frac{\epsilon_0 (F_L - F_U)}{h}$$

This is referred to as ρ_{1-3} or ρ_{1-5} according to the height of the upper mill.

If F_0 is the potential gradient given by the agrimeter, we can obtain :

$$\rho_{0-1} = \frac{\epsilon_0 (F_0 - F_L)}{h_1}$$

where h_1 is the height of the lower mill.

If V_L is the potential at the lower mill, then we can also get :

$$\rho_{0-1} = \frac{2\epsilon_0}{h_1^2} (V_L - h_1 F_L)$$

7. Results in fair weather

Six records were taken on different days in fair weather. The first noticeable result is that fluctuations in space charge are much more obvious as the wind speed increases; this is in general agreement with what is expected. On 5 occasions with westerly winds, the average space charge at 1 — 3 m was found to be negative, but on the sixth occasion, with a northerly wind, it was found to be positive; if domestic fires, etc., in Durham City, to the north of the site, produce positive space charges, this difference can be explained.

It is frequently found that there are corresponding changes in potential gradient and in space charge at 1 — 3 m, but of opposite sign. This result can be explained as follows : if a positively charged ion cloud passes close above the mills, then it will produce a larger increase of potential gradient at the upper mill than at the lower, and, on applying formula (1) of Section 6, this would appear to indicate a negative charge; an actual negative space charge between the mills would produce opposite effects on both mills.

Considering all measurements in fair weather, Fig. 1 is obtained. Such a result could

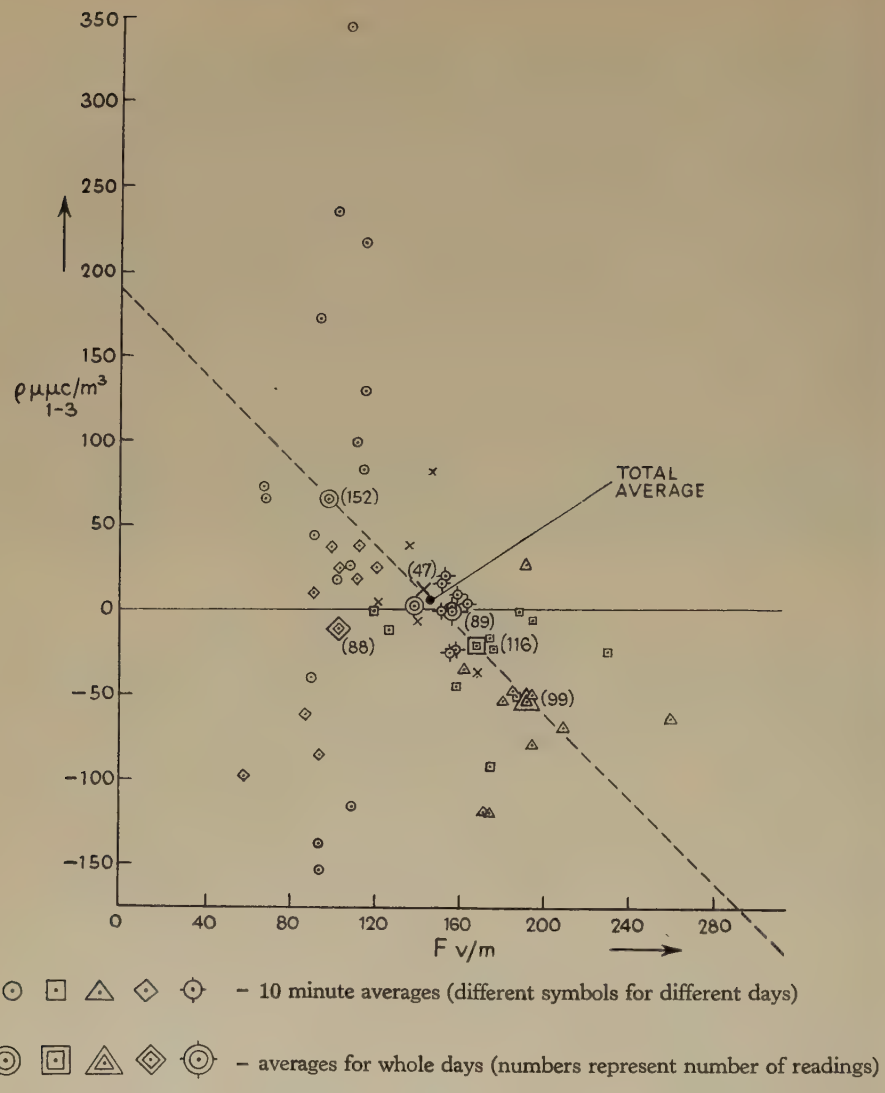

Figure 1. Relation between space charge (1 — 3 m) and potential gradient.

be obtained if there is a difference in ' reduction factor ' for the two mills, but calculation shows that the difference required would be much greater than could occur. Instead, it is suggested that Fig. 1 can be explained in terms of radioactivity of the ground. A negative space charge between 1 m and 3 m can be accounted for by a greater conductivity at 1 m than at 3 m; this might be due to some extra negative ions reaching 1 m but not 3 m. If radioactivity produces ions near the ground, then the negative ions so produced move upwards and affect the conductivity so far as they remain small ions. If the average life of a small ion is assumed to be the same for the different records, then the larger the potential gradient, the farther will small ions travel during their lifetime and so the more will reach up to the 1 — 3 m region; few will get to 3 m so that difference in conductivity between 1 m and 3 m is greater the greater the potential gradient.

The cases in Fig. 1, illustrated by ◇, giving negative space charges for lower potential gradients than apply in the other cases, are from a day in which there had been heavy rain earlier in the day and this may have ' washed out ' condensation nuclei and so increased the average life of small ions.

The above arguments are supported by the results for ρ_{0-1}, which, in general, show a greater negative space charge when ρ_{1-3} shows a less value, and vice versa.

A comparison between the results from the field mills and those from the filtration apparatus can be made only if ρ_{0-1} is used, since the filtration apparatus is at 65 cm. In most cases, there is quite good agreement and this can be taken as verification that the double field mills can be used to measure space charge. On one occasion, however, the filtration apparatus gave an average value of $+ 84$ $\mu\mu$c m^{-3} while the mill measurements gave $- 55$ $\mu\mu$c m^{-3}; this was the same occasion as mentioned above when there had previously been heavy rain, and when it is considered that condensation nuclei had been removed. Therefore the space charge in this case would be largely on small ions, and the induced negative charge (in a positive potential gradient) on the filtration apparatus would attract positive and repel negative small ions. It seems that the filtration apparatus, as used here, might give erroneous results when small ions are exceptionally prevalent.

Further measurements in fair weather would be required to confirm the above suggestions.

8. Results in rain

Results have been obtained under various conditions of rain, and the first important conclusion is that values of the space charge at 1 — 5 m are not much larger than in fine weather unless the rate of rainfall is greater than about 0·1 mm min^{-1}. For rates of rainfall from 0·06 to 0·1 mm min^{-1}, a space charge is found below 1 m but not above. If, as seems probable, the production of space charge is due to splashing, then the critical rate of rainfall may depend upon the nature of the ground at the time.

In heavy rain, the results are consistent with the idea that splashing produces negative ions, and cases occur when there is a negative space charge and a negative potential gradient, contrary to the results of Adkins (1959). The largest values of space charge are of the order of $- 1{,}000$ $\mu\mu$c m^{-3}. Some inconsistencies between the results of the filtration apparatus and the mills may be explained by the prevalence of small ions, as for the fair-weather results.

In one case of a heavy shower, it was found that, after the rain had ceased, a negative space charge appeared at 1 — 3 m; presumably ions that had previously been lower had now risen.

9. Results in snow

The results during snow show space charges of both signs at different times and do not allow of any simple explanation. Since the rate of fall of snowflakes is small, the space charge of the snow itself may contribute appreciably to the total space charge. Maximum values of space charge were about ± 500 $\mu\mu$c m^{-3}.

10. Conclusions

The first conclusion is that the method with double field mills can be used to give space charges; in fair weather, a negative space charge is often found and may be associated with radioactive effects, confirming the ideas that have been put forward in connection with the electrode effect. When small ions are more plentiful than usual, it appears that the filtration apparatus, as used here, may give erroneous results.

In rain, it is found that little space charge is produced until the rate of rainfall becomes high. In such cases, there is a production of negative space charge, presumably by splashing. These results do not agree with the results of Adkins (1959) who found that the sign of the space charge was opposite to that of the potential gradient but only in cases of potential gradients larger than those measured here; nor do they agree with the theory of Smith (1955) who suggested that the negative potential gradient, even in light continuous rain, might be due to effects of splashing.

More complete details of the records and results are contained in a report by the authors (Chalmers and Smiddy 1958).

It seems possible to use the double field mill as a collector for other purposes; for example, if the outside of the filtration apparatus, as used in the present work, were maintained at the potential of its surroundings, instead of at earth potential, it would give more useful results.

Acknowledgments

Thanks are due to fellow workers in the group, particularly to Mr. M. W. Ramsay, and Mr. J. W. Milner, and to the technicians, Mr. C. Maclean and Mr. M. Binks, for much assistance.

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Point-discharge currents from natural and artificial points

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Summary

When point discharge occurs at an earth-connected point, ions of one sign travel to earth through the point while ions of opposite sign travel away in the wind. This stream of ions reduces the potential gradient at the ground to leeward of the point below that to windward. Theory shows that, under suitable conditions, this difference in potential gradient is close to that predicted for a line of charge, even though the line is spread into a cone by turbulent diffusion. Measurements with point discharge from a 34 m mast show good agreement with theory, and this makes it possible to use the difference of potential gradient to determine the point-discharge current through a tree. When applied to a line of trees, seasonal differences are found suggesting smaller currents from trees in leaf than from bare trees. Measurements with a 12 m sycamore tree in full leaf show no point-discharge current even in potential gradients up to 7,000 v/m.

1. Introduction

Point-discharge currents are believed to make an important contribution to the exchange of electricity between clouds and earth, but nearly all the measurements have been made with artificial points, so that it cannot be certain what are the point-discharge currents from natural trees.

Schonland (1928) cut down a small tree and supported it on insulators, but this is not practicable for larger trees and, in any case, such a tree differs from a living tree both in the electrical properties of its wood and in the foliage. Kirkman (1956) attempted to measure the current in a tree by using two conducting bands round the tree at different heights, a number of screws being used to make contact between a band and the conducting part of the tree; the bands were connected through a low-resistance galvanometer, which would then short-circuit the part of the tree. The method did not give satisfactory results, probably because of the contact-potential effects at the screws.

It appears that the best approach to the problem of the current through a tree is to use an indirect method.

2. Outline of method

When point discharge occurs, ions of both signs are produced close to the point; those of the same sign as the potential gradient move into the point and give the point-discharge current, while those of the opposite sign move away in the field and in the wind. Since equal numbers of ions of either sign are produced, it is possible to obtain the point-discharge current if it is possible to measure the current of ions of opposite sign.

Excepting very close to the point, the ions move with the wind (even in a potential gradient of 10,000 v/m, the velocity in the field is only about 1·5 m sec^{-1}); therefore there is no appreciable difference in motion when the ions become large ions rather than small. There is also no effect of recombination of ions, since all that is important is the net excess of charge in a volume.

The current of ions moving away from the point can be recognised by the electrostatic effect at the ground beneath it. For the elimination of the natural potential gradient, it is necessary to measure, simultaneously, the potential gradients at either side of the discharging point in the line of the wind.

3. Theory

The simplest case to consider is that of a line of charge moving away from the discharging point horizontally and passing directly over the measuring instrument. The potential gradients recorded by such an instrument and one placed at the same distance from the discharging point in the opposite direction (upwind) should give a difference of

$$\frac{i}{2\pi\epsilon_0 V h}\left\{1+\frac{d}{(d^2+h^2)^{\frac{1}{2}}}\right\}$$

where h is the height of the point, d the horizontal distance of the measuring site from the base of the point, i the current and V the wind speed (Davis and Standring 1947).

It can easily be shown that, if the measuring site is not immediately below the line of charge, there is a correction which is less than 5 per cent if $h = 34$ m, $d < 45$ m and the angle between the line of charge and the direction of d is less than 10°.

However, the actual conditions are less simple than envisaged in this theory, because the ions do not travel along the same horizontal straight line but are spread by diffusion into a cone. The process at work is turbulent diffusion and the extent to which it operates depends on the existing meteorological conditions, particularly in regard to vertical temperature gradient. The motion of the ions must be very similar to that of smoke emitted from a tall chimney and so it is possible to use an equation derived by Sutton (1947) for that problem. Applied to a source at $x = 0$, $y = 0$, $z = h$, the density of charge at a point (x, y, z) is :

$$Q=\frac{i}{\pi c^2 x^{2-n} V}\exp\left[-\frac{1}{x^{2-n}}\left\{\frac{y^2}{c^2}+\frac{(z-h)^2}{c^2}\right\}\right]$$

where n is a number between 0 and 1, c is a constant, i is the current and V the horizontal wind speed; both n and c depend on the meteorological conditions.

The vertical electric potential gradient at a point $(d, 0, 0)$, due to this cone of ions is then :

$$F=\frac{2}{4\pi\epsilon_0}\int_{-\infty}^{+\infty}\int\int Q\frac{z\,dx\,dy\,dz}{\{(x-d)^2+y^2+z^2\}^{3/2}}$$

with the factor 2 for the image charge.

This integral cannot be evaluated directly, even for $n = 0$ or 1, but it is possible to approximate. For any one value of x, we can consider points distant r from the axis of the cone, and if we take $R = \{h^2 + (x - d)^2\}^{\frac{1}{2}}$, then it is possible to evaluate integrals with respect to y and z in terms of r/R. If $d < \sqrt{3}\,h$, it can be shown that $(r/R)^6$ can be neglected. The integration with respect to x can be carried out only by graphical methods and it is thus found that, with $h = 34$ m, for $d = 10$ m or 45 m, the final result is not significantly different from that for a line distribution of charge, whether the conditions are chosen as large lapse ($c = 0{\cdot}36$, $n = 0{\cdot}2$) or large inversion ($c = 0{\cdot}03$, $n = 0{\cdot}5$), using Sutton's extreme values. It thus appears that atmospheric turbulence has negligible effect on the electric potential gradient produced by the stream of ions from a point of height 34 m provided that d is less than about 50 m, and so the simple theory of Davis and Standring (1947) is applicable.

4. Apparatus for verification of theory

In order to verify the theory, simultaneous measurements were made of the potential gradient at points upwind and downwind of a 34 m mast carrying a discharging point; the point-discharge current through the point and the wind speed were also measured.

For the measurement of the potential gradient, field mills were used, of the type described by Mapleson and Whitlock (1955). In most measurements of potential gradients, it is relative values which are of most interest, but here the absolute values are of importance and therefore it is necessary to carry out calibrations with care. It was found that calibration by the classical method of a stretched wire with a burning fuse was not satisfactory and, instead, one of the mills was placed in a pit with its vanes flush with the surrounding earth; then a large aluminium plate, with a hole for the vanes, was placed on the ground. A second plate was mounted on insulators parallel to, and 40 cm above, the first plate. On applying a potential difference between the plates, the mill would register the signal corresponding to a known potential gradient and, on removing the plate, could be used as a standard for the calibration of other field mills. For day-to-day calibration, the mills could be covered by a close fitting plate and the absolute value of the calibration determined as above. It was concluded that the absolute value of the potential gradient could be determined to an accuracy of ± 5 per cent over a range of 0 to $\pm 3{,}000$ v/m.

For the measurement of the point-discharge current, the same mast and the same point were used as in the work of Kirkman and Chalmers (1957). A similar anemometer was also used, to measure wind speed.

In order to determine the wind direction, a wind-vane was arranged to move a carbon brush over a wire-wound resistance and this acted as a potentiometer giving the wind direction.

Sites for the field mills were arranged at 10 m, 25 m and 41 m, from the mast, along 3 lines at intervals of 20°. Recordings were taken only if the wind direction lay within 10° of one of the lines; at the commencement of recording three mills were placed along the most suitable line if the wind direction was steady and across the lines if changing. A fourth field mill was at 50 m in a direction nearly opposite to the central line mentioned above, which was slightly north of east from the mast.

5. Results for verification of theory

Point discharge at the point above the mast occurs when the potential gradient at the ground exceeds ± 220 v/m (Kirkman and Chalmers 1957). This is not the only condition that must be satisfied before useful records could be taken. The potential gradient at the ground must not exceed $\pm 1{,}300$ v/m as then there occurs point discharge at other objects, e.g., trees. The wind direction must be within 10° of one of the lines of mills. Also fluctuations in wind speed, wind direction and potential gradient must not be too rapid.

From a total recording time of 11 hours, it was possible to obtain 75 values for 41 m, 75 for 25 m and 25 for 10 m. According to the theory, the difference, $E_u - E_d$, between the potential gradients at upwind and downwind sites should be proportional to the ratio i/V, where i is the point-discharge current and V the wind speed. By statistical analysis, the three sets of results were fitted to a formula :

$$E_u - E_d = A\, i/V + B$$

and the values of A and B obtained are shown in Table 1, together with the value of A from theory (theoretically B is zero).

The agreement with theory is seen to be very good and when tested statistically the results are found to be highly significant.

TABLE 1. Values obtained

Distance	A	Standard error of A	B	Calculated value of A
41 m	779	± 28	5	803
25 m	722	± 16	0	720
10 m	602	± 35	6	609

No noticeable differences were found between cases of positive and negative potential gradients. Measurements were made in conditions of various degrees of turbulence ranging from high lapse rate to moderate inversion (for strong inversion, wind speeds were too low and wind direction too variable); in agreement with the theory, no significant difference between the results for different conditions of turbulence could be detected.

Light to moderate rain produced no effect on the results, but in heavy rain effects were obtained which would agree qualitatively with the idea that the ions are 'washed out' by the rain; a quantitative evaluation in terms of the capture of ions by drops according to the theory worked out by Whipple and Chalmers (1944) does not suggest as big an effect as is found; there is possibly some effect of splashing.

6. Application to discharge from trees

The measurements quoted above show that the relation between $F_u - F_d$, i and V is as predicted by theory, so similar measurements for point discharge from a tree could be used to obtain i if the other factors are known.

The first results were concerned with discharge from a line of trees somewhat to the north of the upwind mill. As previously mentioned, the simple theory breaks down when the potential gradient at the ground exceeds $\pm$ 1,300 v/m, and records when this occurred were rejected in discussing the verification of the theory; these records were, however, used in investigating the effect of the line of trees. Assuming that the same result holds for the trees as for the point, namely that the ions produced an effect similar to that of a line of charge, it is possible to calculate the effect of the trees on the different field mills; the effect of the known current through the point at the top of the mast can be allowed for, and, since this current depends upon the value of the undisturbed potential gradient, this can also be deduced. Using these results it was possible to deduce the point-discharge current from the line of trees; plotting this against the undisturbed potential gradient gave a straight line similar to those found by Kirkman and Chalmers (1957). The above procedure was carried out for two sets of observations, one in December 1956 and the other in May 1957, and the results compared. It was found that the slope of the line in December was 3·1 times that in May, although the wind speed was nearly the same. The only difference that might account for this is that in December none of the trees had leaves, but in May, most of them (poplars) were in full leaf, although one (an ash) was still bare; it is suggested that this accounts for the different results. Conditions in which such measurements can be made are rare and it has not been possible to obtain any further similar data.

Since results with a single tree would be simpler than with a line of trees, transportable apparatus was constructed and arrangements were made to obtain measurements on an isolated sycamore tree 12 m high on Durham City Golf Course. Recordings were made on three occasions when it might have been expected that potential gradients would have been large enough for point discharge and on one occasion the value reached 7,000 v/m. The results showed absolutely no indication of point discharge; it is certain that the effect due to a current of 1 μa would have been detected and probably a smaller current, but there was not even the irregularity of the downwind mill that is characteristic of the effect of point-discharge ions (Whitlock and Chalmers 1956; Large and Pierce 1956). These recordings were all taken with the sycamore tree in full leaf, thus agreeing with the suggestion from the results of the line of trees that a tree in full leaf does not give point discharge anything like as readily as a bare tree or an artificial point. It has not yet been possible to obtain other results in similar conditions.

7. Conclusions

The conclusion can be reached that the indirect method used here for determining the point-discharge current through a tree is practicable, though further measurements are needed, preferably with permanent installation of apparatus.

There is a strong indication from the results that a tree in full leaf gives much less point discharge than has previously been thought to be the case. If this is confirmed, it may have important repercussions on the contribution of point-discharge currents to the maintenance of the negative charge on the earth.

Acknowledgments

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Discussion

Prof. P. A. Sheppard (*President*) : Does Dr. Maund's work with the isolated tree indicate that such a tree in leaf gives rise to no significant point-discharge current in a thunderstorm ?

The isolated tree in leaf can be treated as an ellipsoid and the field at its surface computed. Were such computations made and did they show that a point-discharge field could not be reached at the true surface for normal thunderstorm fields ?

Dr. T. W. Wormell : Dr. Maund appears to have proved that the point-discharge current from his tree in summer conditions is considerably smaller than most people would have conjectured. But our estimates of the total ionic current under a thunderstorm does not depend solely on guesses about the contribution of a single tree.

Would Dr. Maund agree that his observations probably indicate that some other process giving a copious supply of ions at the ground was becoming effective (e.g., splashing) while the electric field was still too small to give a significant point-discharge current from his tree ?

Dr. W. C. A. Hutchinson : If we refer to Dr. Maund's straight-line graph* relating some function of point-discharge current with the difference between the upwind and the downwind field-mill outputs, we see three points for periods of heavy rain, where the field-mill output difference seems to be much too low. Dr. Maund said he thought this might be due to splashing. Could it not also have been due to the removal of ions by the rain, thus reducing their horizontal movement ?

I wonder if this effect might be shown up by measurements of rain current density upwind and downwind of the discharging point.

* Shown during presentation of paper at Durham but not reproduced.

Dr. J. M. Stagg : This method of assessing the point discharge from a live tree is ingenious and has given valuable results but it is indirect. Is it not practicable to attack the problem directly by measuring the current that actually flows up and down the tree? Botanists tell us that the flow of nutrient sap is confined to a sheath just under the bark. This layer should therefore be the most electrically conducting part of the whole tree, so that a copper band let into that layer around the tree should be able to drain away most of the up-and-down current flow without killing the tree. I myself started an experiment on those lines at Kew Observatory in 1947 but was unable to continue it.

Mr. J. W. Milner : In answer to the question whether point-discharge current down trees could be measured directly by metal bands in connection with the sapwood, the contact potentials produced by metal connections mask any point-discharge currents which would be measured. However, if mercury contacts are used the contact potentials are quite small and very steady and records of point-discharge currents have been satisfactorily obtained. The mercury contacts are formed by drilling small holes into the tree trunk, inserting glass tubes and filling them with mercury.

As yet there are not enough results available to investigate the behaviour of these point-discharge currents down trees with any degree of thoroughness.

Miss C. M. Botley : I think a study of the type of tree on which St. Elmo's Fire appears might be helpful : it certainly appears on pine forests of Christmas-tree type.

Reply

Dr. Maund and Dr. Chalmers (*in reply, partly communicated*) : In reply to Prof. Sheppard, the work showed that the potential gradient required to produce significant point-discharge currents ($> 1\ \mu a$) is greater than previously thought.

The calculations suggested by Prof. Sheppard were in fact performed and the results showed that, if one accepts Wilson's (1925) estimate of a potential gradient of 20,000 Vm^{-1} as needed to produce point discharge at a typical rough surface, then the potential gradient remote from the tree corresponding to this onset potential gradient at the periphery of the tree amounts to about 7,000 Vm^{-1}, which was the maximum potential gradient recorded in these observations.

Replying to Dr. Wormell, the work reported indicates that the point-discharge current down a typical tree in leaf is less than that measured down an artificial point of similar height and exposure. From the results obtained at Durham it is possible only to make qualitative estimates of the excess positive current to the base of a thundercloud and until quantitative measurements are available it cannot be said whether it will be necessary to look for a fresh source of charge near the ground.

In reply to Dr. Hutchinson, calculations were carried out on the removal of ions from the downwind ion stream by rain, according to the process of ion capture suggested by Wilson (1929) and worked out by Whipple and Chalmers (1944). The effect can be appreciable only if the raindrops carry charges of opposite sign to the ions present, i.e., if the rain current and point-discharge current are of the same sign, a condition which, as seen in the results of Simpson (1949), is rare. Measurements of the type suggested would provide an interesting test of Wilson's theory.

Replying to Miss Botley, the survey suggested would be very interesting but difficult to undertake if only visual observations can be made.

It must be emphasized that the results that we have presented are only of a preliminary nature and indicate that there is still considerable work on the subject to be undertaken. The results cannot be significantly interpreted in terms of point-discharge current density but they do indicate the difficulty of extrapolating from a single artificial point to a natural tree of similar height.

A second question, the possible effects of foliage on a point-discharge currents also emerged, but more work is needed before quantitative results are available.

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The performance of a Gunn-Bellani radiation integrator

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SUMMARY

An alcohol-filled Gunn-Bellani radiation integrator, exposed at Rothamsted from June to December 1958, gave weekly distillation totals that were a linear function of short-wave radiation on a horizontal surface above a limit of 500 cal cm^{-2} $week^{-1}$. Laboratory experiments with both alcohol- and water-filled models reveal a threshold radiation intensity which can be estimated from the properties of the distilling fluid and the partial pressure of air within the instruments. Distillation is negligible below the threshold, but above, it is proportional to the excess radiation. Because the relation between absorbed heat and radiation incident on a horizontal surface depends on solar altitude and on the proportions of direct and diffuse radiation, it is difficult to predict field response from laboratory data.

1. FIELD PERFORMANCE

The behaviour of a commercially available Gunn-Bellani radiation integrator in tropical conditions has been the subject of a recent study by Pereira (1959) who reports satisfactory correlation between the amount of water distilled from the spherical receiver of the instrument, and short-wave radiation on a horizontal surface measured with standard equipment. There are two reasons why a correlation in the tropics should be superior to that obtained elsewhere. First, the effective ambient temperature, largely determined by soil temperature, is relatively independent of season. Second, seasonal changes in the relation between the radiation received by spherical and horizontal surfaces have a minimum value at the equator.

In an attempt to assess their usefulness in the latitude of the British Isles, the manufacturers kindly loaned two integrators to Rothamsted Experimental Station, one filled with water and the other with ethyl alcohol, and daily distillation of the latter has been compared with the radiation received by a Moll-Gorzynski thermopile (Kipp and Zonen) as recorded on a self-balancing potentiometer (Messrs. George Kent). The potentiometer has a full-scale sensitivity of 9 mV and with a suitable shunt reads directly in cal cm^{-2} min^{-1}. Mechanical integration gives radiation totals on a counter unit.

The result of the comparison is shown in Fig. 1 for the period June to December 1958. The integrator (No. 68A) contained 38·5 cm^3 alcohol and this was all distilled by an input of about 400 cal cm^{-2}. On days when total radiation exceeded this figure no useful record was obtained and such days have been omitted from the weekly totals. At the lower end of the scale, there was negligible distillation after the end of October, when radiation was less than 500 cal cm^{-2} $week^{-1}$ and the data obtained in November and December were omitted when calculating the regression line and correlation coefficient. The equation of the line is

$$D = 84{\cdot}0\,S - 49 \qquad (r^2 = 0{\cdot}94)$$

where D is the total distillate (cm^3 $week^{-1}$)

S is the total radiation on a horizontal surface (kcal cm^{-2} $week^{-1}$)

r is the correlation coefficient.

The scatter of daily values concealed by averaging over weekly periods is shown by the two lines on Fig. 1 representing the envelope of daily points plotted on the same diagram with both scales reduced by a factor of seven. Variations from the mean response are so large as to make daily readings of no practical value. The water-filled instrument, which was not installed until the end of September, gave no useful correlation in the following four months even for weekly totals.

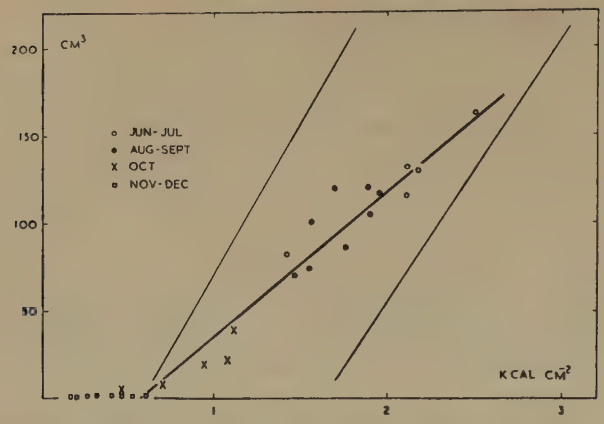

Figure 1. Weekly distillation totals (cm^3) for an alcohol-filled integrator and solar radiation on a horizontal surface ($kcal\ cm^{-2}$). Parallel lines are the envelope of daily points plotted with scale units reduced by a factor of seven.

2. Laboratory performance

A curious feature common to the present response and to that reported by Pereira is the existence of a radiation threshold below which distillation is negligible. Further study of this was undertaken in the laboratory using a fan-cooled 500-watt pearl bulb as a source of radiation, measured with a solarimetric thermopile. The integrator was mounted on a wooden framework surrounded by a transparent plastic cylinder so that the burette could be easily read while protected from air currents. Geometrically, the arrangement was the same as that in the field when the integrator, sunk in the soil, is surrounded by a metal cylinder. The calibration was repeated on the laboratory roof thus giving two ambient temperatures, one near 22°C and the other near 5°C. Results for the alcohol-filled instrument No. 68A and for the instrument of identical construction containing water (No. 53) are shown in Fig. 2.

The following hypotheses attempt to explain why a change of ambient temperature altered the intercept of the calibration curve on the radiation axis without changing its slope.

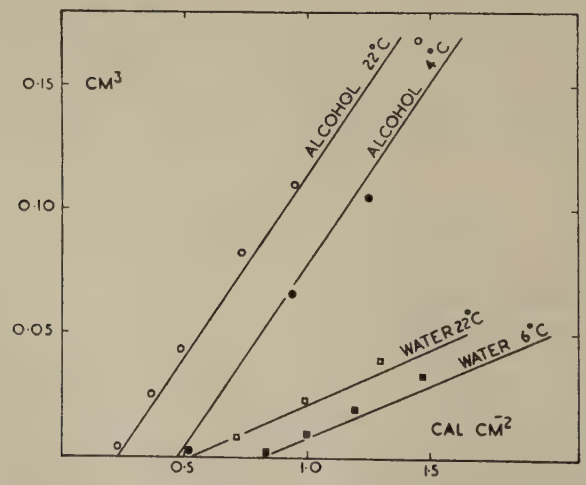

Figure 2. Response of water- and alcohol-filled integrators in the laboratory, used to find S_c (squares and circles) : and response given by Eqs. 1 and 2 (straight lines).

(i) For a given integrator at given ambient temperature, there is a critical receiver temperature T_c below which diffusion of vapour from receiver to condenser is imperceptibly slow.

(ii) It can be shown that the diffusion rate is proportional to $\ln p_2/p_1$ where p_1 and p_2 are respectively the partial pressures of air at the evaporating and condensing surfaces. If the short-wave radiation absorbed by the receiver is sufficient to raise its temperature to T_c, effectively the boiling point, p_1 tends to zero and the mechanism of distillation changes from negligible diffusion to substantial mass flow.

(iii) The temperature T_c is uniquely related to a critical amount of absorbed radiant energy. When radiation exceeds this limit there is no sensible increase of temperature because virtually all excess energy is removed from the receiver as latent heat of evaporation, i.e., efficiency is close to 100 per cent (Courvoisier and Wierzejewski 1954).

An estimate of critical temperature can be obtained by assuming that when mass flow occurs, the saturated vapour pressure e_c of the liquid in the receiver is equal to the total gas pressure in the condenser (It can be shown from Poiseuille's formula that a very small total pressure difference is sufficient for observed transfer rates). The condenser pressure is the sum of the saturated vapour pressure e of the liquid at condenser temperature assumed equal to ambient, and the partial pressure of air in the condenser which can be estimated from pV/V_2 where V is the total volume of air in the system, V_2 is the volume of air in the condenser, and p is the initial (uniform) partial pressure of air. The assumption that a uniform partial pressure of air exists in the condenser during distillation is justified by the observation that condensation takes place near the top of the condenser tube.

For the alcohol instrument at 22°C, p is 20 mm of mercury, e is 49·5 mm, and V and V_2 are approximately 110 and 80 cm^3 with little alcohol in the condenser.

Hence $$e_c = pV/V_2 + e$$

from which $$T_c = 29{\cdot}7°\text{C}.$$

Equating energy received above the critical level with the heat required to vaporize D cm^3 of fluid with density ρ and latent heat L in appropriate units

$$(S - S_c)\,A = \rho LD \quad . \quad . \quad . \quad . \quad (1)$$

where S is short-wave radiation per unit horizontal surface with critical value S_c and A is defined as the effective area of receiver which will be a function of source/receiver geometry. From the response of the alcohol instrument at 22°C the effective area for the present calibration was 25·5 cm^2.

The heat transfer coefficient h for heat loss from the receiver by long-wave radiation, convection and conduction, can now be estimated from

$$S_c\,A = h\,(T_c - T_0) \quad . \quad . \quad . \quad . \quad (2)$$

where T_0 is ambient temperature and S_c is determined from the points of Fig. 2. Plotting $S_c\,A$ against $(T_c - T_0)$ (Fig. 3) gives a straight line through the origin with slope $h = 0{\cdot}013$ cal sec^{-1} °C^{-1}, which is only slightly greater than the transfer coefficient for long-wave radiative exchange alone, approximately 0·011 cal sec^{-1} °C^{-1} for a sphere of diameter 5 cm at 20°C.

Appropriate values of $(T_c - T_0)$ were found from vapour-pressure tables and from the partial pressure of air within the instruments. For the alcohol-filled model, the figure of 20 mm internal air pressure was given by the makers. The corresponding figure for the water-filled model was found by manipulating the instrument until only a few cubic centimetres of water remained in the condenser tube, the tip of which was then immersed in a beaker of water. By adjusting the temperature of this water, reproducible measurements of the internal boiling point were obtained. Internal air pressure was then calculated as the difference between saturation vapour pressure at the boiling point and at ambient temperature, and was found to be 23 mm.

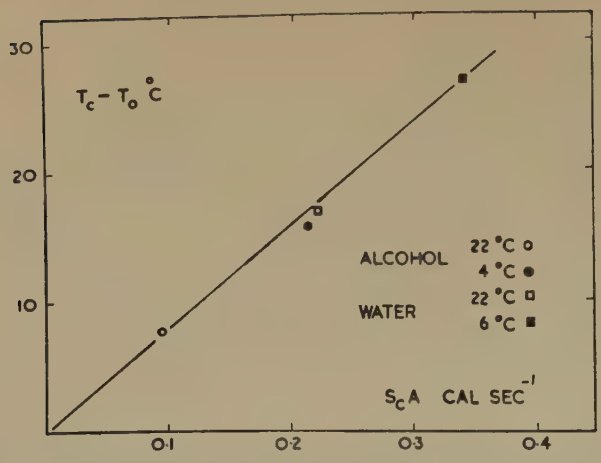

Figure 3. Radiation $S_c A$ required to maintain receiver at critical temperature T_c when ambient temperature is T_0. The slope of the line gives the total heat transfer coefficient for radiative, convective, and conductive losses as 0·013 cal sec^{-1} °C^{-1}.

Given the effective area and the internal air pressure, response can now be predicted from Eqs. 1 and 2 assuming 100 per cent efficiency above the threshold (Fig. 2). This reverses the procedure of Gunn, Kirk and Waterhouse (1945) and of Gunn and Yeo (1951) who calculated efficiency from instrumental parameters and source/receiver geometry, finding a variation from 100 per cent in the laboratory to 70 per cent in the field.

3. Conclusions

The field response of the present alcohol-filled instrument is sufficiently linear to be useful in agricultural and ecological research for eight months of the year in climates similar to that of the British Isles, but the existence of a temperature-dependent radiation-threshold is an undesirable feature of the response which could seriously mislead comparison of radiation-regimes. The original instruments described by Gunn, Kirk and Waterhouse, and those which have been examined in great detail by Courvoisier and Wierzejewski, were evacuated to pressures of the order of 10^{-5} mm and showed no threshold, presumably because mass flow could begin at a very low excess temperature of the receiver. In practice, this might be dangerous if small ambient temperature gradients between receiver and condenser induced significant distillation at rates independent of radiation. Optimum air pressure may lie between Gunn's value and the 20 to 23 mm of the instruments described here.

Even knowing the laboratory response to radiation in a collimated beam, it would be difficult to predict field response as a function of radiation on a horizontal surface because the effective area of the receiver is a function of solar altitude and of the relative intensities of direct and diffuse radiation. In the present calibration with an uncollimated laboratory source, response above the threshold was 145 cm^3 alcohol per kcal cm^{-2} compared with only 84 cm^3 per kcal cm^{-2} in the field.

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Some long-term means from *The Phenological Reports* (1891-1948) of the Royal Meteorological Society

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Summary

The Phenological Reports 1891-1948 of the Royal Meteorological Society are discussed; interpolated values in floral records have been re-calculated, and 58-yr district and British Isles weighted means, and annual differences from these, have been calculated for 11 flowering events. 20-yr means and differences have been similarly calculated for a further 14 events, and the former approximately corrected to the 58-yr means. For 11 main flowers in *The Phenological Reports*, the mean difference in flowering dates between south and north Britain was 21 days later for a distance of some 400 mi farther north. Extremes of earliness and lateness have been tabulated for the British Isles weighted means for these 11 flowers over the years 1891-1948. Extreme earliness averaged 17·7 days and extreme lateness 17·1 days, but while extremes of earliness were fairly constant throughout the year or possibly became a little greater for summer than for spring flowers, extremes of lateness were much greater for spring than for late summer flowers. These differences are directionally in keeping with the finding of Hawke (1941) that in the winter months abnormally low temperatures are much more extreme than abnormally high values, while in summer the abnormally high values are more extreme than the exceptionally low. The standard deviations for the annual values of the British Isles weighted means are also given, and averaged 7·0 days for the 11 main flowers.

1. Introduction

The phenology of plant flowering is a subject which has at times aroused considerable popular enthusiasm, as witnessed by the fact that in 1931, 606 observers sent in a total of 17,000 records for *The Phenological Reports* of the Royal Meteorological Society. Phenological data, however, also find an essential place in modern botany and plant ecology. They are found in every number of the *Biological Flora of the British Isles* (various authors 1941-1958), and on almost every page of the *Flora of the British Isles* (Clapham, Tutin and Warburg 1957) in which the months of flowering are given for each main species; and in the study and use of such data it is of interest to consider what level of accuracy can be reached in phenological studies and what variations there are in Britain with different districts and from year to year. By far the most accurate survey of the phenology of plant flowering in this country has been that presented in *The Phenological Reports* (1877-1948) mentioned above. Table 1 illustrates some of these results. The close regularity of the differences from the 35-yr averages seen in the last column indicates, among other things, the small random errors involved in the means of the large number of observations used.

In 1891 these surveys took a form which was destined to remain almost unaltered until 1947, a span of 57 years. In an average year there were about 200 observers sending in records from 11 meteorological districts of Britain, and in the final tabulations (each year until 1936) the dates for the flowering of particular species reported by all observers within a single district were summed and means were given, as well as (in the later years) 35-yr means for the years 1891-1925. Means for England and Wales and for the British Isles were also given.

TABLE 1. MEAN DATES (DAY OF YEAR) FOR THE FLOWERING OF 14 PLANTS IN 1931, AND DIFFERENCES FROM THE AVERAGE OF THE 35 YEARS 1891–1925, (ENGLAND AND WALES)

	1931	Average 35 years	Difference from average
Hazel (*Corylus avellana*)*	43	39	+ 4
Coltsfoot (*Tussilago farfara*)	65	60	+ 5
Wood-anemone (*Anemone nemorosa*)	92	88	+ 4
Blackthorn (*Prunus spinosa*)	104	98	+ 6
Garlic hedge-mustard (*Sisymbrium alliaria*)	118	114	+ 4
Horse-chestnut (*Aesculus hippocastanum*)	134	129	+ 5
Hawthorn (*Crataegus oxyacantha*)	141	133	+ 8
White ox-eye (*Chrysanthemum leucanthemum*)	154	148	+ 6
Dog-rose (*Rosa canina*)	164	159	+ 5
Black knapweed (*Centaurea nigra*)	189	180	+ 9
Harebell (*Campanula rotundifolia*)	193	189	+ 4
Greater bindweed (*Convolvulus sepium*)	193	191	+ 2
Devil's-bit scabious (*Scabiosa succisa*)	218	216	+ 2
Ivy (*Hedera helix*)	277	270	+ 7

* The names used here and elsewhere in this paper are unaltered from *The Phenological Reports*.

The last *Phenological Report* published was that for 1947, though the data for 1948 were compiled by Mr. J. Tyssen-Gee in manuscript form. None of the district, England and Wales, or British Isles means was, however, given for the years from 1937 onwards because of a break in the manner of presentation of *The Phenological Report* at that date, and it is now clear that a good deal more information can be obtained from the data than has so far been published. In particular, this break in presentation can be rectified and the whole of the data tabulated as a continuous series; and also 58-yr means can be calculated to replace the 35-yr means previously given. This has now been done, and in addition, the phenology of 14 other plants recorded in *The Phenological Reports* of 1929-1948 has been considered, and their 20-yr means corrected to 58-yr means, so that 58-yr means, or approximations to them, are made available for 25 phenological events. For studying differences between one year and another the British Isles means have the advantage over district means that owing to the larger number of observers, the random errors were reduced; but in the present paper a British Isles weighted mean has been introduced in which, for reasons which will be advanced, the random errors are further reduced.

2. THE CALCULATIONS OF 58-YR MEANS AND WEIGHTED MEANS FOR 25 PLANTS IN BRITAIN

11 flower species were used consistently throughout the 58-yr period 1891-1948 of the Royal Meteorological Society's surveys, and these species are prefixed * in Table 2. In the last 20 years (1929-1948) of the surveys, data were given for a number of other phenological events in addition, and from these 13 flowering times and one date of leafing have been selected. (These are unstarred items in Table 2). These two sets of data must be discussed separately, but in the Table they are for convenience presented together, in the order in which they appear during the year.

Observers sent in their reports of dates of first flowering from each of the 11 meteorological districts given in Table 3. (Records from the Channel Islands, Isle of Man, Orkney and Shetland Islands were not used in these computations). The dates sent in were expressed as the day of the year, where 1 January = Day 1 and, except in leap years, 31 December = Day 365. In leap years, 29 February becomes Day 60 and subsequent dates are increased by 1. Following the convention of *The Phenological Reports*, 31, 30, December of a previous year are Days — 1, — 2 etc.

TABLE 2. 58-YR WEIGHTED MEANS FOR FLOWERING TIMES IN BRITAIN FOR 11 MAIN PLANTS MARKED *, AND 20-YR WEIGHTED MEANS ADJUSTED TO 58-YR MEANS FOR 14 SUBSIDIARY PLANTS

	Day of year
Christmas rose (*Helleborus niger*)	− 9·5
Winter aconite (*Eranthis hiemalis*)	21·0
Snowdrop (*Galanthus nivalis*)	25·1
*Hazel (*Corylus avellana*)	41·3
Yellow crocus (*Crocus aureus*)	41·3
Lesser celandine (*Ranunculus ficaria*)	59·4
*Coltsfoot (*Tussilago farfara*)	62·4
Almond (*Amygdalus communis*)	79·0
*Wood-anemone (*Anemone nemorosa*)	88·3
Horse-chestnut (leafing) (*Aesculus hippocastanum*)	100·4
Red-currant (*Ribes rubrum*)	102·4
*Garlic hedge-mustard (*Sisymbrium alliaria*)	113·7
Bird-cherry (*Prunus padus*)	122·1
Purple lilac (*Syringa vulgaris*)	126·1
*Horse-chestnut (flowering) (*Aesculus hippocastanum*)	128·5
Laburnum (*Cytisus laburnum*)	132·6
*Hawthorn (*Crataegus oxyacantha*)	133·2
*White ox-eye (*Chrysanthemum leucanthemum*)	149·5
Elder (*Sambucus nigra*)	155·1
*Dog-rose (*Rosa canina*)	159·3
Madonna-lily (*Lilium candidum*)	188·6
*Greater bindweed (*Convolvulus sepium*)	190·6
*Harebell (*Campanula rotundifolia*)	190·8
Autumn crocus (*Colchicum autumnale*)	248·9
*Ivy (*Hedera helix*)	270·8

TABLE 3. METEOROLOGICAL DISTRICTS USED, AND WEIGHTING FACTORS

		District	Weighting factor
A	(8)	SW. England and S. Wales	7
B	(10)	S. Ireland	2
C	(5)	SE. England	10
D	(4)	Mid. England	9
E	(3)	E. England	6
F	(7)	NW. England and N. Wales	5
G	(9)	N. Ireland	3
H	(6)	W. Scotland	1
I	(2)	NE. England	4
J	(1)	E. Scotland	2
K	(0)	N. Scotland	1

The first step in the present work was to calculate the annual values of district mean flowering times for each of the above flowers and districts for the years 1937-1948, for which summations had not previously been made. For the 11 flowers (starred), for the period 1891-1948 there should then ideally have been $58 \times 11 \times 11 = 7{,}018$ district means in all, but in 309 instances, through lack of observers, records were missing. In some cases interpolated values had been supplied by the compilers of *The Phenological Reports*, but the methods of interpolation were not the same throughout the years. All interpolations were therefore re-calculated on a uniform basis.

For the method of interpolation adopted it is necessary to refer to a further stage in the computations, the calculation of British Isles weighted means for each flower for each year, and also for the whole 58-yr period. For this purpose weighting factors were applied to district means. In explanation of the method adopted it must be observed that a straight mean of all individual records will obviously have less observational error than the average of the 11 district means, which themselves depend on very different numbers of observers and therefore have different variances. To take such an average of all observer-records year after year is objectionable, however, because the numbers of observers in districts are changing; and if there were a northerly drift, for example, in observers this would be reflected as a lateness in the mean of the records. If, however, the annual district means are multiplied by the average number of observers (averaged over all the years together) in each respective district, and such products for all districts are summed and then divided by the total average number of observers, the effect of such a drift is checked, while at the same time a mean is obtained which approaches a straight mean of all records both in actual value and in accuracy, and is decidedly better in the latter respect than the average of the 11 district means. Such a mean has the added advantage that the greater part of the weighting entering into the final figures represents returns with a somewhat smaller geographical scatter than was the case when all districts were given equal weighting.

To calculate the weighted means, the straight annual means for each of the 11 districts were therefore multiplied by the weighting factors given in Table 3. These factors are approximately proportional to the total number of observers reporting from each district over the whole period 1891-1948, but for convenience of subsequent calculation, have been rounded to give whole numbers. They add up to 50. For each plant and year the 11 district means, multiplied by these weightings, summed, and divided by 50 gave the annual values of the British Isles weighted means. The mean of the 58 values of these then gave the 58-yr British Isles weighted mean for each plant.

Straight 58-yr district means for each of the 11 flowers in each of the 11 districts were also calculated. The interpolation for a particular species in a particular district and year was then obtained by correcting the 58-yr district mean for that plant by the difference of the annual value of its British Isles weighted mean from the 58-yr British Isles weighted mean. The use of these interpolated values then slightly altered the district and weighted means, and calculations were continued by the process of successive approximations until no further alterations occurred.

The 58-yr means for each of the 11 districts and for the 11 main flowers are shown in Table 4, while the 58-yr weighted means for the 11 flowers (starred items) are given in Table 2. The differences from these latter means of the annual values of the British Isles weighted means are given in Table 5.

TABLE 4 58-YR (1891-1948) DISTRICT MEANS FOR 11 METEOROLOGICAL DISTRICTS AND 11 MAIN PLANTS

	District										
	A	B	C	D	E	F	G	H	I	J	K
Hazel	38	35	36	43	36	44	47	50	48	56	56
Coltsfoot	57	58	60	64	62	63	66	67	65	74	76
Wood-anemone	85	77	83	90	88	91	86	98	94	102	103
Garlic hedge-mustard	108	110	110	113	111	120	111	125	119	130	131
Horse-chestnut	122	122	125	129	126	131	129	138	137	145	142
Hawthorn	129	130	130	133	130	136	136	142	138	149	150
White ox-eye	145	144	143	150	146	154	154	159	158	165	166
Dog-rose	156	158	155	159	155	162	164	165	166	171	168
Greater bindweed	185	192	186	191	188	193	200	196	196	204	201
Harebell	193	186	188	192	189	190	196	195	192	195	195
Ivy	265	264	268	269	267	274	274	277	279	288	292

TABLE 5. DIFFERENCES FROM 58-YR WEIGHTED MEANS FOR 11 MAIN PLANTS

	Hazel	Coltsfoot	Wood-anemone	Garlic hedge-mustard	Horse-chestnut	Hawthorn	White ox-eye	Dog-rose	Greater bindweed	Harebell	Ivy
1891	+ 2·9	+ 4·1	+ 8·9	+ 11·0	+ 14·1	+ 14·3	+ 8·3	+ 11·4	+ 11·7	+ 0·8	+ 12·9
1892	+ 0·5	+ 5·7	+ 6·2	+ 5·1	+ 9·7	+ 7·5	+ 2·8	+ 6·3	+ 4·6	+ 1·0	+ 7·1
1893	− 0·1	− 2·2	− 7·5	− 9·4	− 16·4	− 18·3	− 16·5	− 20·9	− 20·6	− 12·0	− 17·4
1894	− 10·6	− 6·4	− 6·2	− 10·6	− 15·9	− 14·8	− 6·1	− 0·2	+ 5·4	− 1·6	+ 6·2
1895	+ 30·3	+ 14·7	+ 6·5	+ 4·9	− 1·6	+ 4·7	− 2·9	− 1·4	− 7·4	− 18·8	− 6·9
1896	− 10·1	− 3·2	− 7·0	− 6·0	− 9·0	− 9·7	− 8·4	− 10·6	− 10·5	− 5·0	− 6·9
1897	− 3·0	− 7·4	− 7·7	− 2·3	− 0·4	+ 0·9	− 0·7	− 0·9	− 2·8	− 3·8	− 2·1
1898	− 15·1	− 13·0	− 6·8	+ 0·4	+ 2·9	+ 1·2	+ 1·6	+ 4·2	+ 7·1	+ 3·7	+ 0·9
1899	− 3·5	− 2·0	+ 5·2	+ 3·8	+ 6·2	+ 6·0	+ 2·8	+ 3·4	+ 1·7	+ 0·5	− 0·5
1900	+ 7·9	+ 11·0	+ 10·3	+ 7·5	+ 6·4	+ 8·7	+ 4·4	+ 5·4	+ 5·8	+ 2·1	+ 6·9
1901	+ 4·3	+ 4·9	+ 7·9	+ 6·7	+ 2·8	+ 6·6	− 0·5	+ 0·2	− 4·6	− 3·0	− 1·1
1902	+ 0·4	+ 7·4	+ 1·3	+ 3·0	+ 3·0	+ 8·0	+ 7·5	+ 12·0	+ 9·5	+ 3·1	+ 10·4
1903	− 3·8	− 6·5	− 8·7	− 8·9	+ 6·5	+ 0·3	+ 0·8	+ 1·6	+ 8·3	+ 4·7	+ 4·8
1904	+ 1·4	+ 7·1	+ 3·2	+ 4·9	+ 3·1	+ 6·4	+ 2·1	+ 1·1	+ 8·9	+ 3·0	+ 1·0
1905	− 2·5	+ 0·2	− 4·9	− 2·1	+ 1·9	+ 1·7	− 0·1	+ 0·3	+ 1·3	+ 0·4	− 1·3
1906	− 8·7	− 0·9	− 5·8	− 3·8	+ 5·4	+ 4·3	+ 1·1	+ 4·8	+ 4·2	+ 1·3	− 1·5
1907	+ 6·9	+ 10·6	+ 1·3	+ 1·1	+ 0·8	+ 4·2	+ 2·4	+ 6·7	+ 15·2	+ 8·9	+ 5·9
1908	− 0·5	+ 0·8	+ 3·3	+ 7·8	+ 9·2	+ 9·9	+ 3·4	+ 3·5	+ 5·4	− 2·4	+ 4·1
1909	+ 2·3	+ 12·8	+ 8·4	+ 6·9	+ 1·7	+ 4·6	+ 1·6	+ 4·3	+ 9·8	+ 1·8	+ 8·0
1910	+ 2·5	+ 0·6	− 2·1	− 0·6	+ 4·7	+ 5·3	+ 0·6	+ 0·9	+ 6·8	+ 2·2	+ 0·6
1911	− 1·3	− 3·8	− 2·1	+ 1·0	+ 0·2	+ 0·4	− 3·3	− 5·6	− 1·7	− 5·6	− 8·0
1912	− 6·6	− 1·4	− 6·5	− 6·7	− 9·8	− 9·6	− 6·2	− 9·6	− 1·3	− 2·9	− 1·4
1913	− 2·5	− 14·6	− 9·3	− 9·3	+ 0·1	− 1·7	− 2·3	− 2·1	+ 2·0	− 1·5	− 4·1
1914	− 4·2	− 10·0	− 6·1	− 4·8	− 8·6	− 9·4	− 5·3	− 6·8	− 3·8	− 7·1	− 6·8
1915	− 3·1	− 1·6	+ 2·4	+ 3·8	+ 2·3	+ 2·3	− 2·3	+ 0·9	+ 0·1	− 1·5	− 0·9
1916	− 13·7	− 8·2	+ 4·8	+ 2·4	+ 2·3	+ 0·8	+ 0·8	+ 2·2	+ 11·1	+ 4·3	+ 0·1
1917	+ 18·4	+ 16·8	+ 20·2	+ 13·9	+ 8·0	+ 10·4	+ 2·9	+ 3·0	− 1·8	− 5·4	− 3·7
1918	− 6·5	− 7·9	− 7·9	− 2·8	+ 0·7	− 2·9	− 2·8	− 7·4	− 3·4	− 3·2	− 4·0
1919	+ 4·2	+ 5·1	+ 6·3	+ 4·6	+ 4·4	+ 4·2	− 2·4	− 2·7	− 3·6	− 3·9	− 6·6
1920	− 10·3	− 9·1	− 9·6	− 14·0	− 8·7	− 11·0	− 4·8	− 7·1	− 1·0	+ 0·2	+ 2·8
1921	− 12·4	− 17·9	− 14·8	− 13·1	− 10·8	− 12·7	− 8·9	− 10·9	− 8·9	− 0·2	− 5·0
1922	− 3·3	− 2·9	+ 7·5	+ 9·3	+ 7·5	+ 5·5	+ 2·8	− 0·5	− 3·8	+ 1·6	+ 2·0
1923	− 5·3	− 13·8	− 10·5	− 10·7	− 2·5	− 3·3	− 1·9	+ 3·0	+ 6·9	+ 6·8	+ 7·0
1924	− 3·6	+ 2·9	+ 3·4	+ 7·0	+ 7·9	+ 7·7	+ 3·4	+ 4·4	+ 4·8	+ 4·9	+ 5·9
1925	− 4·1	− 9·9	− 2·6	+ 3·2	+ 5·3	+ 4·5	+ 1·7	+ 1·2	+ 1·1	+ 1·0	+ 0·2
1926	− 7·2	− 8·8	− 12·9	− 9·4	− 7·5	− 12·4	− 0·6	− 2·4	+ 3·4	+ 2·1	− 0·2
1927	− 1·7	− 1·2	− 4·1	− 5·9	− 2·4	− 3·4	− 0·8	+ 0·5	+ 4·3	+ 5·2	+ 4·0
1928	− 1·1	− 6·6	− 4·1	− 2·8	− 0·4	− 4·2	− 0·2	+ 0·8	+ 5·1	+ 6·5	+ 2·0
1929	+ 14·2	+ 10·7	+ 4·6	+ 4·0	+ 6·2	+ 5·7	+ 2·9	+ 4·5	+ 4·6	+ 0·2	+ 2·8
1930	+ 0·8	+ 3·6	+ 6·3	+ 3·0	+ 4·3	+ 5·9	+ 2·7	+ 3·6	+ 1·7	− 0·1	+ 1·1
1931	+ 2·6	+ 3·2	+ 3·8	+ 3·9	+ 5·7	+ 7·6	+ 5·1	+ 5·5	+ 4·2	+ 1·9	+ 7·6
1932	− 1·7	− 6·0	+ 1·3	+ 6·8	+ 9·9	+ 10·8	+ 9·3	+ 9·5	+ 6·5	+ 3·4	+ 4·0
1933	− 3·3	− 1·5	− 3·6	− 5·5	− 7·7	− 6·1	− 3·2	− 5·0	− 5·6	− 1·9	− 5·1
1934	− 6·9	+ 3·7	+ 5·8	+ 3·3	+ 3·2	+ 4·2	+ 1·8	+ 0·5	− 2·3	− 3·1	− 0·8
1935	− 12·0	− 8·5	− 2·7	− 3·0	− 3·6	− 3·9	+ 0·9	+ 2·3	+ 2·0	+ 0·7	+ 0·9
1936	+ 4·0	+ 7·7	+ 2·1	+ 5·5	+ 4·5	+ 4·8	+ 4·4	+ 4·1	+ 2·9	+ 0·9	+ 1·3
1937	− 1·0	+ 0·9	+ 4·8	+ 1·9	+ 2·7	+ 2·2	+ 2·2	+ 0·7	− 1·4	+ 1·8	− 1·1
1938	− 4·6	− 6·8	− 8·5	− 9·8	− 9·3	− 14·1	− 2·2	+ 0·4	− 0·9	+ 2·4	+ 4·7
1939	+ 0·9	− 5·7	+ 0·8	− 0·9	− 2·8	− 2·9	+ 0·8	− 2·2	− 5·5	+ 1·4	+ 1·1
1940	+ 18·8	+ 11·6	+ 7·5	+ 5·2	− 0·3	+ 0·7	− 0·3	− 1·9	− 12·1	− 5·1	− 3·6
1941	+ 7·5	+ 8·6	+ 9·7	+ 10·0	+ 15·8	+ 16·4	+ 12·7	+ 11·8	+ 1·1	+ 2·3	+ 2·2
1942	+ 20·5	+ 16·9	+ 9·9	+ 5·7	+ 0·3	+ 4·6	+ 2·3	+ 4·6	− 5·8	− 1·4	− 0·6
1943	− 4·3	− 7·0	− 4·6	− 8·9	− 13·3	− 15·4	− 8·2	− 11·1	− 13·3	+ 1·0	− 4·7
1944	− 9·4	− 2·4	+ 1·5	− 0·6	− 4·5	− 5·3	+ 0·8	− 1·8	− 3·6	+ 5·3	− 3·7
1945	+ 2·2	− 2·1	− 8·9	− 11·1	− 19·7	− 19·1	− 9·2	− 13·3	− 12·9	− 3·4	− 9·5
1946	− 1·7	+ 0·6	− 0·4	− 2·0	− 10·2	− 9·3	+ 0·6	− 1·5	− 6·4	+ 5·5	− 0·1
1947	+ 34·5	+ 25·7	+ 13·5	+ 11·8	+ 2·8	+ 7·0	+ 3·3	+ 0·3	− 9·1	− 4·7	− 5·8
1948	+ 1·6	+ 2·7	− 3·3	− 4·6	− 6·5	− 9·0	− 1·8	− 6·6	− 12·2	+ 6·7	− 4·0

The computational procedure for the 14 unstarred items in Table 2, for which 20 years of records only (1929-1948) were available, must now be given. Firstly, annual values of the district means for each plant, 20-yr district means, and annual and 20-yr British Isles weighted means were calculated, again using the weighting factors of Table 3. The procedure for interpolated values, and adjusting by the method of successive approximations was then carried out as before. It seemed desirable, however, to adjust the final 20-yr means to the full 58-yr interval, to be in keeping with the rest of the data, and to do this 20-yr means and weighted means (1929-1948) were calculated for the 11 main plants first discussed. Differences, in days, between 20-yr and 58-yr weighted means for these 11 plants (values of which are given by Jeffree 1957) were used to obtain a smoothed curve from which could be read the corrections appropriate for the 14 supplementary plants. These corrections, applied to the 20-yr means and weighted means were :

	Days		Days
Christmas rose	− 1·4	Red-currant	− 1·3
Winter aconite	− 2·6	Bird-cherry	− 0·5
Snowdrop	− 2·8	Purple lilac	− 0·4
Yellow crocus	− 3·1	Laburnum	− 0·2
Lesser celandine	− 2·9	Elder	+ 0·5
Almond	− 2·2	Madonna-lily	+ 1·3
Horse-chestnut (leafing)	− 1·4	Autumn crocus	+ 1·0

Table 6 gives the adjusted district means for these plants, and Table 7 shows the differences of the annual values of the British Isles weighted means from the approximations to the 58-yr weighted means. The latter means are given (against the unstarred plants) in Table 2.

TABLE 6. 20-YR (1929-1948) DISTRICT MEANS FOR 11 METEOROLOGICAL DISTRICTS AND 14 SUBSIDIARY PLANTS, ADJUSTED TO 58-YR MEANS

	District										
	A	B	C	D	E	F	G	H	I	J	K
Christmas rose	− 15	− 25	− 6	− 10	− 6	− 11	− 5	− 21	− 10	+ 8	− 16
Winter aconite	18	7	22	23	19	23	15	26	24	29	29
Snowdrop	18	15	25	31	26	24	18	24	30	34	32
Yellow crocus	32	27	38	46	41	42	38	47	49	56	58
Lesser celandine	44	40	58	68	61	63	41	68	69	83	76
Almond	65	76	75	83	79	77	88	90	88	94	95
Horse-chestnut (leafing)	96	91	99	102	100	102	97	103	107	110	110
Red-currant	97	97	101	104	101	104	97	108	108	112	116
Bird-cherry	117	122	120	122	121	126	121	121	126	134	133
Purple lilac	120	124	122	128	124	128	130	131	129	143	140
Laburnum	128	126	130	135	131	133	132	134	135	148	145
Elder	149	148	151	156	152	158	157	165	162	173	168
Madonna-lily	184	188	184	187	184	193	194	202	194	204	204
Autumn crocus	248	250	246	247	245	253	254	250	253	253	250

3. THE VARIABILITY OF PHENOLOGICAL DATES IN BRITAIN WITH LATITUDE AND ALTITUDE

The differences between district means for latest and earliest districts for the 11 main plants (Table 4) vary between 10 days for harebell and 28 days for ivy, with an overall mean of 21 days. The earliest districts are SW. England, S. Ireland and SE. England respectively, the latter for midsummer flowers only, and the latest almost always E. or N. Scotland. Thus, broadly, between about Bristol and Aberdeen, or some 400 mi in a S. to N. direction, flowering times have become 21 days later, or one day for every 19 mi (3·7 days per degree of lat. N.). This may be compared with the general statement of Hopkins (1938) that flowering dates are delayed approximately 4 days for each 1° of lat. N.

TABLE 7. DIFFERENCES FROM THE APPROXIMATIONS TO 58-YR WEIGHTED MEANS FOR 14 SUBSIDIARY PLANTS

	Christmas rose	Winter aconite	Snowdrop	Yellow crocus	Lesser celandine	Almond	Horse-chestnut (leafing)	Red-currant	Bird-cherry	Purple lilac	Laburnum	Elder	Madonna-lily	Autumn crocus
1929	− 5·7	+ 9·5	+ 8·4	+14·3	+10·4	+13·9	− 1·7	+ 2·6	+ 6·0	+ 7·2	+ 6·5	+ 1·4	+ 4·8	+ 1·7
1930	− 0·7	− 4·1	− 2·6	− 0·2	+ 1·1	+ 6·0	+ 1·3	+10·4	+ 9·4	+ 7·6	+ 6·9	+ 3·8	+ 1·7	− 1·5
1931	+ 0·1	− 1·0	− 1·5	+ 1·2	− 2·2	+10·0	+ 3·5	+ 8·2	+11·2	+ 8·7	+ 8·0	+ 5·5	+ 5·9	+ 1·5
1932	− 7·1	− 9·1	− 8·8	−10·7	−16·3	+ 1·5	+ 5·3	+ 9·9	+12·8	+12·8	+ 9·6	+ 9·8	+ 6·4	− 1·6
1933	− 1·4	− 3·1	+ 0·2	+ 3·7	− 3·8	− 3·6	− 6·6	− 2·7	− 3·4	− 4·4	− 5·2	− 4·3	− 7·7	−10·8
1934	+11·8	+ 2·8	− 1·3	+ 1·0	+10·9	+10·1	+ 5·5	+ 2·7	+ 5·4	+ 3·1	+ 2·7	+ 3·9	− 2·3	− 1·5
1935	− 6·0	−12·2	−12·7	−10·5	−15·8	−17·4	− 4·8	− 1·9	+ 1·2	− 1·3	− 3·9	− 2·2	− 0·7	− 0·1
1936	+ 4·3	+ 3·2	+ 1·6	+ 9·5	+ 6·5	+ 5·3	+ 1·5	+ 0·2	+ 5·0	+ 6·1	+ 4·0	+ 5·2	+ 0·1	+ 1·2
1937	+ 1·2	− 4·9	− 5·6	− 4·7	− 3·6	− 4·0	+ 7·2	+ 2·1	+ 3·6	+ 3·6	+ 1·8	+ 1·9	− 0·9	− 2·7
1938	+ 2·9	− 0·3	− 3·1	− 4·6	− 3·5	− 9·1	−10·3	−11·5	− 8·1	−15·6	− 7·9	− 4·7	− 1·3	− 3·1
1939	+ 1·7	− 1·0	− 0·3	− 4·6	− 9·6	− 2·6	+ 1·9	− 1·3	− 0·6	− 2·5	+ 0·3	− 2·4	− 1·2	+ 1·0
1940	+ 8·2	+21·1	+20·7	+17·1	+15·4	+12·6	+ 6·3	+ 4·4	+ 3·1	+ 3·5	+ 2·7	− 1·0	− 8·3	− 3·2
1941	+ 0·1	+ 6·9	+ 5·2	+ 8·4	+15·7	+12·2	+15·0	+ 9·5	+13·4	+16·0	+16·1	+12·5	+ 3·4	+ 0·2
1942	+ 5·3	+17·8	+19·0	+24·6	+19·2	+18·2	+ 6·8	+ 7·3	+ 2·8	+ 3·6	+ 5·4	+ 3·9	+ 2·6	+ 0·5
1943	− 2·3	− 2·1	− 0·9	− 6·0	− 8·4	−15·2	− 5·5	− 5·1	−15·0	−13·1	−13·5	−12·6	− 9·1	− 4·6
1944	+ 5·2	− 3·4	− 5·2	− 8·8	+ 0·6	− 3·6	+ 4·2	+ 1·6	− 7·1	− 4·0	− 4·7	− 4·9	− 3·4	− 0·7
1945	− 3·1	+ 5·4	+ 8·1	+ 4·7	+ 2·0	− 5·6	− 8·9	−13·1	−19·2	−16·7	−18·7	−12·7	−10·6	− 5·5
1946	+ 0·7	+ 0·3	− 1·5	− 3·9	+ 2·9	− 2·2	− 2·2	− 6·3	− 9·2	− 7·5	− 6·5	− 6·3	− 0·7	+ 5·5
1947	+ 7·2	+21·4	+32·0	+28·5	+26·9	+27·7	+11·0	+11·5	+ 7·5	+ 7·5	+ 6·1	+ 1·8	− 0·5	− 0·1
1948	+ 5·1	+ 4·0	+ 4·7	+ 2·7	+ 8·8	− 9·3	− 1·8	− 3·0	− 9·4	− 6·8	− 6·5	− 8·1	− 3·5	+ 4·5

Hopkins showed that across major continental areas lines of equal phenological dates (' isophanes ' or ' isophenes ') deviate from lines of latitude by about 1° lat. to each 5° long., the dates being earlier towards the west of such land masses; and he noted the relationship between this inclination of the isophenes relative to the lines of latitude and the very approximately similar inclination of sea-level reduced isotherms of annual mean temperatures. The isotherms are, however, much more variable in other areas (particularly near the sea), and while they run very roughly WNW. to ESE. across Britain they are by no means regular in this direction. Furthermore, the summer and winter isotherms run in very different directions. The effects on the isophenes of the temperatures of the different months are not completely known. Average floral isophenes for Britain over the 35 years 1891-1925 were, however, given by Clark, Margary and Marshall (1927) and show great variability in direction.

Hopkins found, in addition, a phenological delay of 1 day for each 100 ft increase in altitude. Smith (1938), in his statistical examination of some data from *The Phenological Reports*, found a significant regression of flowering time for hawthorn on altitude ($b = 0{\cdot}0130 \pm {\cdot}0035$), for stations grouped in areas of about 19 miles square, showing that flowering is on the average delayed 1·3 days for each 100 ft increase in altitude. For hazel he obtained almost the same regression coefficient ($b = 0{\cdot}0127 \pm {\cdot}0099$), but in this case the value was not significant. No statistical investigation is known to have been made so far on the possibly systematic variation of such coefficients for species of flowers appearing at different times of the year, but observations at Aberdeen suggest that spring flowers may be delayed more than summer flowers.

4. THE MAGNITUDES OF EXTREME PHENOLOGICAL DEVIATIONS THROUGHOUT THE YEAR

Table 8 shows values of extreme positive and negative deviations for each of the 11 main plants as shown in the British Isles weighted means. It will be noted that extreme earliness can occur somewhat equally for all the flowers, i.e., throughout the year, though

TABLE 8. Extreme values for the differences in days between annual values of British Isles weighted mean flowering times and the 58-yr (1891-1948) weighted means, and standard deviations of annual values

	Extreme negative deviations	Year	Extreme positive deviations	Year	Standard deviation of annual values
Hazel	− 15·1	1898	+ 34·5	1947	9·6
Coltsfoot	− 17·9	1921	+ 25·7	1947	8·7
Wood-anemone	− 14·8	1921	+ 20·2	1917	7·3
Garlic hedge-mustard	− 14·0	1920	+ 13·9	1917	6·9
Horse-chestnut	− 19·7	1945	+ 15·8	1941	7·6
Hawthorn	− 19·1	1945	+ 16·4	1941	8·4
White ox-eye	− 16·5	1893	+ 12·7	1941	4·8
Dog-rose	− 20·9	1893	+ 12·0	1902	6·2
Greater bindweed	− 20·6	1893	+ 15·2	1907	7·2
Harebell	− 18·8	1895	+ 8·9	1907	4·6
Ivy	− 17·4	1893	+ 12·9	1891	5·4
Mean	− 17·7		+ 17·1		7·0

spring values are slightly lower than summer values, but extreme lateness can be much greater in early spring than in summer. While for the first three spring plants extreme positive deviations are much greater than negative, for the subsequent eight plants the reverse is true, and towards the end of the year (harebell and ivy) this reversal is very marked.

Hawke (1941) in a paper on the frequency distribution through the year of abnormally high and low daily mean temperatures at Greenwich showed, for the period 1841-1936, the number of occasions when the daily mean temperatures were at least 15°F above or below the average. During the months November to March values were frequently below the average to this extent, but only very occasionally 15° above the average, while from April to October, they were often 15° above, but never this much below. The above phenological findings would be broadly in keeping with the influence of such temperature anomalies.

The overall means for extreme negative and extreme positive phenological deviations (− 17·7 and + 17·1 days respectively) are approximately equal.

The last column of Table 8 shows the standard deviations of annual values of the British Isles weighted means for the several plants. These average 7·0 days, and there is an indication of higher values for the first spring plants and a reduction in the deviation towards late summer.

5. Discussion

Smith (1938) has referred to the difficulty of circumventing a wish that flowering dates might be categorically stated to within a day, and he was particularly concerned to study the extent and sources of variability in some of the data of *The Phenological Reports*. He reached the conclusion that ' The standard deviations of observations in restricted localities appear to be about five to seven days for late spring and summer plants, and rising to about thirteen days for early spring and autumn flowering plants.' No greater validity than this should, therefore, be attributed in general to a single observation of a flowering time; but in the means of, for example, 25 or 250 observations the random errors may be reduced approximately to one-fifth or one-sixteenth respectively of those of single observations, and so it becomes possible for some of the results presented above to have a validity of a considerably higher order, represented often by errors of only a day or two, and in some cases even less.

The values indicated in this paper give a background against which other shorter-erm statements of flowering times and their fluctuations may reasonably be set. Plant ecologists, however, are often interested in the phenology of other phases of plant develop-nent as well as flowering, but in this connection it may be of value to compare the differences given for horse-chestnut (leafing) in Table 7 with those for adjacent flowering events, particularly for red-currant flowering, when it is seen that there is little in the dates for his leafing event to distinguish them from those of a nearly synchronous flowering event. Thus this survey of the phenology of flowering, while not necessarily giving exactly what might have been found if quite different phenological events had been selected, may nevertheless give a very fair guide to what may be expected of other phenological events taking place at least in those parts of a plant which are above ground and exposed, as flowering shoots usually are, to air and sun.

In addition to their botanical applications, these data reflect the meteorological conditions of the country and, in a way similar to a tabulation of mean temperatures of each month, give a picture of the character of the year. Hopkins (1938) has been quoted above as finding relationships between isophenes and isotherms, while Smith (1938) reached the conclusion that ' Temperature appears to be the dominating meteorological factor determining flowering dates,' and Grainger (1939) has given correlation diagrams for flowering of coltsfoot (*Tussilago farfara*) in the Midlands of England for 1902-1934 against temperatures of individual months. Other correlations could undoubtedly be calculated from the longer runs of data now presented, and in this connection the extensive records compiled by Manley (1953) of monthly mean temperatures from 1698-1952 of ' Central England ' are of interest, because the geographical centre of the weighted means of the present paper is also basically ' Central England,' and the annual figures given in Tables 5 and 7 are set out in a form similar to Manley's tabulations, so that comparisons of these two sets of data are easily possible. Such comparisons cannot be taken very far in this paper, but it is, for instance, readily shown that the dates for hazel show a strongly marked negative regression on the January temperatures, while the corresponding regres-sion for ivy on September temperatures is far less pronounced.

Acknowledgments

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551.524.31

The diurnal temperature variation during the polar night

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SUMMARY

It is well known that if special types of days are considered separately, systematic diurnal temperature variations may be found to exist at polar stations even during the dark season. In the present paper it is shown that, in all probability, this phenomenon is of trivial nature and that a perfectly simple explanation of its origin is possible.

1. INTRODUCTION

There are some indications that even during the dark season a slight, systematic diurnal temperature change may exist in polar regions. One of the explanations suggested is that the diurnal pressure oscillation may bring about a corresponding variation of the wind, which again may influence the air temperature (Sverdrup 1933). However, for most stations such effects, if any, are difficult to trace, as, owing to the shortness of the observation period, the masking influence of the much greater unsystematic fluctuations cannot be sufficiently reduced. It is true that the mean temperatures for the individual observation hours generally turn out to have a fairly smooth diurnal course, but this may be due to the persistence tendency of the temperature alone, and is hence a feature of little conclusive value in this connection. In most cases the results of a comparison between the data for different winter seasons at the same station, or between the data of different stations, are not completely unambiguous with respect to the character of the diurnal temperature march.

There is, however, another feature of the diurnal temperature variation during the polar night that is much more clearly in evidence, and which, consequently, has been the subject of much discussion. If we, instead of considering the days of the dark season as a whole, study special types of days separately, very systematic diurnal changes of the temperature may appear. For several polar stations it is found that, on an average, days with a clear sky and/or light to moderate winds have their lowest temperatures during daytime, and the highest temperatures during the night. For overcast and/or windy days the conditions are reversed; here the shape of the diurnal variation is the same as that found during summer-time. It does seem difficult to find a reasonable physical explanation of this phenomenon. Simpson (1919) in his classical work on Antarctic meteorology writes : " I have spent more time over this problem than over any other discussed in this work ", but " the investigation has led only to negative results " (p. 72). And hitherto no satisfactory physical explanation has been put forward (cf., Court 1951).

2. DISCUSSION

On the basis of the character of the diurnal temperature variations just mentioned, it is tempting to suggest that there is no need for a physical explanation at all. This suggestion is based on the following facts :

We know that in polar regions during winter-time, days with low wind speeds are generally characterized by lower temperatures than days with high wind speeds, corresponding, *i.a.*, to the existence of inversion and non-inversion conditions, respectively. However, since the temperature as well as the wind speed and cloud amount, change continuously from the one type of days to the other, the temperature on the coldest days (i.e., those with the lowest wind speeds) tends, on an average, to be highest at the beginning and at the end of the day. Conversely, on the warmest days (high wind speeds) the temperature tends to be lowest at these times of the day. This reasoning also applies to the relation between clear and overcast days, the effect being heightened by the fact that there is usually a positive correlation between wind speed and cloud amount.

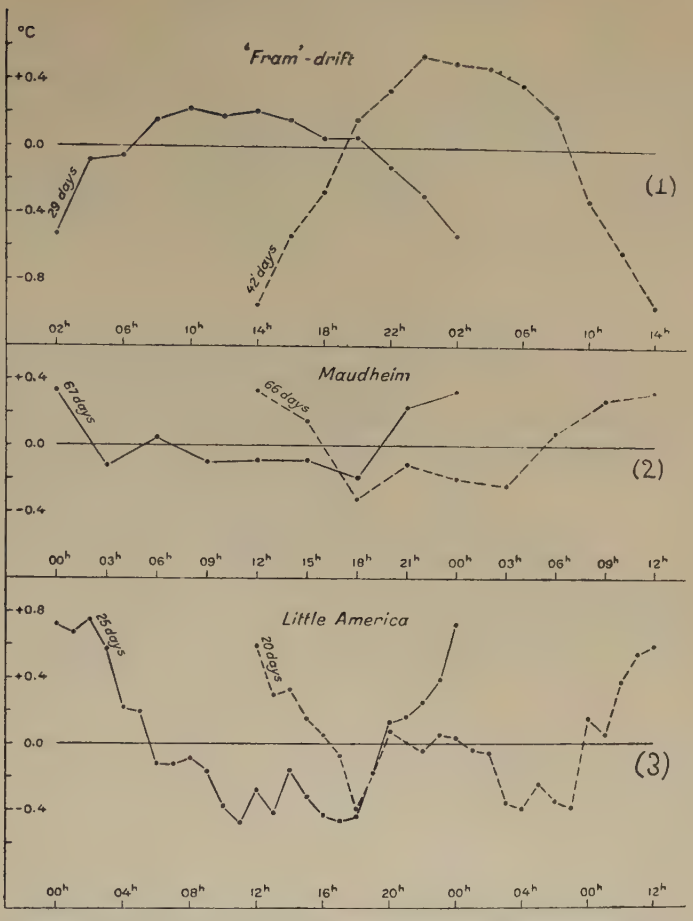

Figure 1. Diurnal variation of the hourly mean temperatures during the dark season. (1) Days with cloud amount above 3/10 ('Fram'-drift); (2) days with wind speed less than 17 kt (Maudheim); (3) days with wind speed less than 15 mi hr^{-1} and cloud amount less than 5/10 (Little America). Full lines: days starting at 00 hr, broken lines: days starting at 12 hr.

Examples of the 'automatic' diurnal variations produced in the above-suggested manner are given in Fig. 1. For the periods when the sun is below the horizon we have plotted the diurnal temperature variations (1) for days with cloud amounts above 3/10 at all observation hours (the 'Fram'-drift); (2) for days with wind speeds less than 17 kt at all observation hours (Maudheim), and (3) for days with wind speed less than 15 mi hr^{-1} and cloud amounts less than 5/10 at all observation hours (Little America). The data for the 'Fram'-drift and for Little America are based on tables given by Mohn (1905) and by Grimminger and Haines (1939). The Maudheim data are based on the observations of the Norwegian-British-Swedish Antarctic Expedition, 1949-1952. The examples are chosen at random. The limits for cloud amount and wind speed are fixed in such a way that a reasonably large number of days is obtained. We have selected the days in two different ways: in the first group the days start, as normally, at 00 hr, in the second group they start at 12 hr. It appears that for one and the same station the general shape of the curves is approximately the same for both groups, and, furthermore, that the temperature variations agree fully with those to be expected on the basis of the effects referred to above. Actually, a negative trend is present (i.e., the mean temperature is higher at the start than at the end of the 24-hr period) in all cases where the course of the curves is concave (upwards). In the cases with a convex course the trend is positive. This may be due to a lag of the temperature changes with respect to the changes of the wind speed and the cloud amount. Differences in the rapidity of the increase and decrease, respectively, of the two latter elements may also play a certain part. The values in the diagram are, in the usual way, corrected for this non-cyclic variation (Lamont's correction).

3. Conclusions

It seems, on the whole, reasonable to conclude that the systematic diurnal temperature variations found for special types of days during the polar night owe their existence more to a statistical bias than to physical processes. Generally speaking, whenever elements subject to a persistence tendency are concerned, analyses of diurnal variations for selected samples of days should be carried out with great care. Not infrequently, systematic variations that may easily be explained as the 'automatic' result of the manner in which the material is classified, are made the subject of involved interpretations.

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Determination of the straight line of best fit to observational data of two related variates when both sets of values are subject to error

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SUMMARY

The general equation to a straight line of best fit to observational data of two related variates x and y is obtained by minimizing the general expression $\sum_{i=1}^{n} (y_i - ax_i - b)^2 \phi(a)$ involving the departures of both variates from the line. It is shown that the only expression for $\phi(a)$ which produces consistent results with change of unit is $\phi(a) =$ constant. a^k, where k depends on the relative errors (e_x, e_y) in measurements of the two variates.

The line of best fit is shown to be

$$\frac{y - \bar{y}}{s_y} = c \frac{x - \bar{x}}{s_x}$$

where c is obtained from the equation $(k + 2)c^2 - 2(k + 1)rc + k = 0$, in which r is the coefficient of correlation between x and y, and k is a given function of e_x and e_y. The usual regression lines and line with a slope equal to the geometric mean of the slopes of the two regression lines, arise as special cases of the general equation. Examples are given to show general application.

1. INTRODUCTION

When analysing meteorological data statistically we occasionally have a sample set of pairs of values of data of two related variates X, Y, which when plotted do not lie exactly along a straight line but to which we require to fit a straight line. The customary method is to determine the parameters of the line by the method of least squares applied to the differences from the line for the variate that is regarded as dependent, i.e., the dependent variate only is regarded as being subject to error. This method gives two lines – the usual regression lines – depending on which variate is regarded as dependent. However, it is often the case that the values of both variates are subject to error, in which event the customary approach is of little assistance in determining a single line of best fit. Greenwood and Yule (1915) used the principal axis as an empirically good solution and Gumbel (1954) on heuristic grounds has used the line with slope equal to the geometric mean of the slopes of the two regression lines. The author, however, is unaware of a general solution of the problem.

In the following the method of least squares is applied to a general expression allowing for errors of both variates and a corresponding general equation to the line of best fit is obtained.

2. SINGLE LINE OF BEST FIT

(a) *Choice of criterion*

Suppose we have a set of pairs of values of the two related variates : $(X_1, Y_1), (X_2, Y_2), \ldots, (X_i, Y_i) \ldots, (X_n, Y_n)$ which are assumed to be plotted as points on a scatter diagram. Let the respective values in terms of the plotted distances be $(x_1, y_1), (x_2, y_2), \ldots, (x_i, y_i), \ldots, (x_n, y_n)$

where

$$x_i = \alpha \frac{X_i}{S_X}, \qquad y_i = \beta \frac{Y_i}{S_Y}$$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i, \qquad \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

$$S_X = \left[\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2\right]^{\frac{1}{2}}, \qquad S_Y = \left[\frac{1}{n} \sum_{i=1}^{n} (Y_i - \bar{Y})^2\right]^{\frac{1}{2}}$$

and α and β are arbitrary constants. Let the equation to the straight line to be fitted to the points be $y = ax + b$, where a and b are constants. As shown in Fig. 1 let A_i represent the point (x_i, y_i) and let lines drawn through A_i parallel to the x and y axes meet the line $y = ax + b$ in

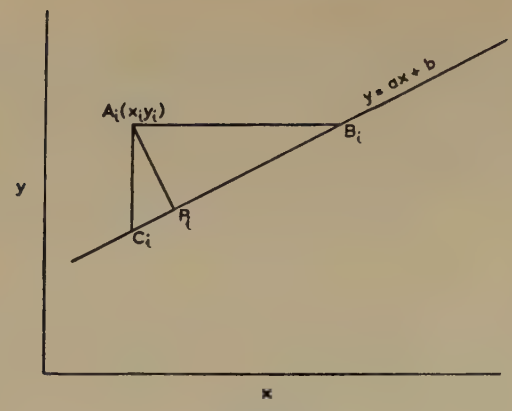

Figure 1.

the points B_i and C_i respectively. Let the foot of the perpendicular form A_i on to the line $y = ax + b$ be at P_i.

The departure of the point A_i from the line $y = ax + b$ can be measured in various ways. For instance, by distances such as $A_i B_i$, $A_i C_i$, $A_i P_i$ or $B_i C_i$. In terms of x_i, y_i, a and b these distances are respectively :

$$(y_i - ax_i - b)/a, \quad y_i - ax_i - b, \quad (y_i - ax_i - b)/(a^2 + 1)^{\frac{1}{2}} \quad \text{and} \quad (y_i - ax_i - b)(a^2 + 1)^{\frac{1}{2}}/a$$

These are all particular cases of the general form $(y_i - ax_i - b)\theta(a)$ where $\theta(a)$ is a function of a alone. We shall now consider the departure as being represented by this general form. If we determine a and b by applying the method of least squares to such departure it means choosing a and b so that $\sum_{i=1}^{n} (y_i - ax_i - b)^2 \theta^2(a)$ is a minimum.

Without loss of generality we can take $\theta^2(a) = \phi(a)$ so that we require to minimize $\sum_{i=1}^{n} (y_i - ax_i - b)^2 \phi(a)$.

(b) *General solution*

Minimizing $\sum_{i=1}^{n} (y_i - ax_i - b)^2 \phi(a)$ with respect to a and b gives

$$(a^2 s_x^2 + s_y^2 - 2a r s_x s_y)\phi'(a) + 2(a s_x^2 - r s_x s_y)\phi(a) = 0 \quad . \quad . \quad (1)$$

and

$$\bar{y} - a\bar{x} - b = 0$$

where

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$s_x = \left[\frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2\right]^{\frac{1}{2}}, \qquad s_y = \left[\frac{1}{n}\sum_{i=1}^{n} (y_i - \bar{y})^2\right]^{\frac{1}{2}}$$

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n s_x s_y} \quad \text{and} \quad \phi'(a) = \frac{d}{da}\phi(a).$$

For the fitted line to give consistent results whatever the scale adopted we require that the slope of the fitted line as given by a should vary directly with the related scales of measurement of the variates. But the values s_x and s_y vary directly with the respective scales of measurement of the variates so that we require a to vary directly with s_y/s_x. If $\lambda = s_y/s_x$ then we require that $a = c\lambda$ where c is independent of λ. Substituting for s_y in equation (1) gives

$$(\psi(c\lambda) + 2)c^2 - 2r(\psi(c\lambda) + 1)c + \psi(c\lambda) = 0 \quad . \quad . \quad . \quad (2)$$

where

$$\psi(c\lambda) = \psi(a) = a\phi'(a)/\phi(a).$$

If $k = \psi(a) \neq -2$ then $c = \dfrac{(k+1)\,r \pm [1-(k+1)^2(1-r^2)]^{\frac{1}{2}}}{k+2}$ so that if c is constant under changes of scale then so is k and so is $a\phi'(a)/\phi(a)$. In consequence

$$\log \phi(a) = k \log a + \text{constant}$$

and

$$\phi(a) = \text{constant}.\ a^k.$$

The required function to be minimized is therefore

$$\sum_{i=1}^{n} (y_i - ax_i - b)^2 a^k \qquad . \quad . \quad . \quad . \quad (3)$$

and the line of best fit is

$$\frac{y - \bar{y}}{s_y} = c\,\frac{x - \bar{x}}{s_x} \qquad . \quad . \quad . \quad . \quad . \quad (4)$$

where c is given by the expression

$$(k+2)\,c^2 - 2r\,(k+1)\,c + k = 0. \qquad . \quad . \quad . \quad (5)$$

We have

$$c = 1/r \text{ when } k = -2 \qquad . \quad . \quad . \quad . \quad (6)$$

and

$$c = \frac{(k+1)\,r \pm [1-(k+1)^2(1-r^2)]^{\frac{1}{2}}}{k+2} \text{ when } k \neq -2, \qquad . \quad . \quad (7)$$

where the sign of the radical in (7) is taken to be the same as that of r. c is real for all values of r when $-2 \leqslant k \leqslant 0$.

We are concerned only with the cases $0 < |r| < 1$ for when $r = 0$ there is no single line of best fit and when $r = \pm 1$ there is an exact correspondence between the two variates. We note that all possible lines pass through the sample mean $(\bar{x}, \bar{y})$ and lie between those given by $k = 0$ and $k = -2$.

Values of c corresponding to various values of k and positive values of r are given in Table 1 and illustrated, for c in the range 0 to 2, in Fig. 2. When r is negative the values of c are identical in absolute magnitude with those corresponding to the positive values of r, but have negative sign.

It may be noted that by expanding the r.h.s. of (7) it can be shown that to a first approximation

$$c = r - k\,(1-r) \text{ when } -1 \leqslant k \leqslant 0 \qquad . \quad . \quad . \quad (8)$$

TABLE 1. VALUES OF c CORRESPONDING TO VARIOUS VALUES OF k (AS GIVEN BY EXPRESSIONS (6) AND (7)) FOR POSITIVE VALUES OF r

k	r* 0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
0·0	0·10	0·20	0·30	0·40	0·50	0·60	0·70	0·80	0·90	1·00
− 0.1	0·28	0·34	0·41	0·49	0·57	0·65	0·73	0·82	0·91	1·00
− 0·2	0·38	0·43	0·49	0·56	0·62	0·69	0·77	0·84	0·92	1·00
− 0·3	0·46	0·51	0·56	0·62	0·67	0·73	0·80	0·86	0·93	1·00
− 0·4	0·54	0·58	0·63	0·67	0·72	0·77	0·83	0·88	0·94	1·00
− 0·5	0·61	0·65	0·69	0·73	0·77	0·81	0·86	0·90	0·95	1·00
− 0·6	0·68	0·71	0·75	0·78	0·81	0·85	0·88	0·92	0·96	1·00
− 0·7	0·76	0·78	0·81	0·83	0·86	0·89	0·91	0·94	0·97	1·00
− 0·8	0·83	0·85	0·87	0·89	0·90	0·92	0·94	0·96	0·98	1·00
− 0·9	0·91	0·92	0·93	0·94	0·95	0·96	0·97	0·98	0·99	1·00
− 1·0	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
− 1·1	1·09	1·08	1·07	1·06	1·05	1·04	1·03	1·02	1·01	1·00
− 1·2	1·20	1·18	1·15	1·13	1·11	1·08	1·06	1·04	1·02	1·00
− 1·3	1·32	1·28	1·24	1·20	1·17	1·13	1·10	1·06	1·03	1·00
− 1·4	1·46	1·40	1·34	1·28	1·23	1·18	1·13	1·08	1·04	1·00
− 1·5	1·63	1·54	1·46	1·38	1·30	1·23	1·17	1·11	1·05	1·00
− 1·6	1·86	1·72	1·60	1·49	1·39	1·29	1·21	1·13	1·06	1·00
− 1·7	2·16	1·96	1·78	1·62	1·48	1·36	1·25	1·16	1·07	1·00
− 1·8	2·63	2·30	2·03	1·80	1·61	1·44	1·30	1·19	1·09	1·00
− 1·9	3·55	2·92	2·43	2·05	1·76	1·54	1·36	1·22	1·10	1·00
− 2·0	10·00	5·00	3·33	2·50	2·00	1·67	1·43	1·25	1·11	1·00

* When r is negative, values of c are as tabled but have negative sign.

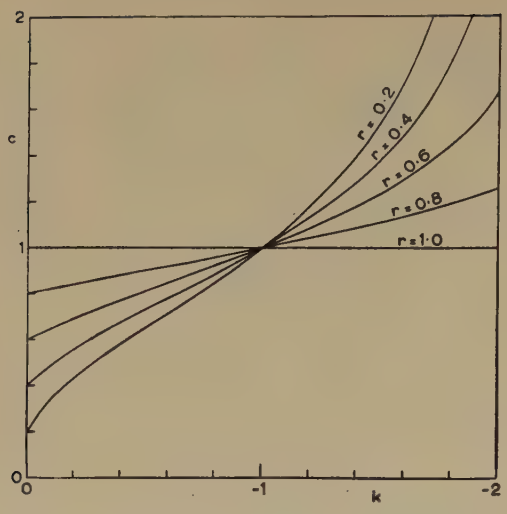

Figure 2. Variation of c with k for various values of r.

(c) *Determination of k*

From expressions (6) and (7) we have :

(a) when $k = 0$, $c = r$ and the line of best fit is the regression line of y on x

(b) when $k = -2$, $c = 1/r$ and the line of best fit is the regression line of x on y,

and (c) when $k = -1$, $c = +1$ when r is positive,

and $c = -1$ when r is negative,

so that the line of best fit has a slope equal to the geometric mean of the slopes of the two regression lines.

If expression (3) is given in terms of the distances $A_i B_i$ $[= (y_i - ax_i - b)/a]$ and $A_i C_i$ $[= y_i - ax_i - b]$, it becomes

$$\sum_{i=1}^{n} (A_i C_i)^2 (A_i C_i / A_i B_i)^k \quad \text{where } -2 \leqslant k \leqslant 0 \quad . \quad . \quad . \quad (9)$$

or

$$\sum_{i=1}^{n} (A_i C_i)^2 (A_i B_i / A_i C_i)^{k'} \quad \text{where } 0 \leqslant k' = -k \leqslant 2 \quad . \quad . \quad (10)$$

Now consider

$$\sum_{i=1}^{n} (A_i C_i)^2 (A_i B_i / A_i C_i)^{k'} \quad \text{where } 0 \leqslant k' \leqslant 1 \quad . \quad . \quad . \quad (11)$$

This is the sum of the weighted squares of the departures of the y values from the fitted line; the individual weighting factors being $(A_i B_i / A_i C_i)^{k'}$. Now k' depends on the relative accuracy of the two variates, as $k' = 0$ corresponds to the x variate being considered to be free from error and $k' = 1$ corresponds to the x and y variates having equal errors. If e_x represents the error of the x variate (measured in units of α) and e_y the error of the y variate (measured in units of β) we may take $k' = e_x/e_y$.

For the range $-2 \leqslant k \leqslant -1$ we take $k'' = 2 + k$, then the expression to be minimized becomes

$$\sum_{i=1}^{n} (A_i B_i)^2 (A_i C_i / A_i B_i)^{k''} \quad \text{where } 0 \leqslant k'' \leqslant 1 \quad . \quad . \quad . \quad (12)$$

which is of similar form to (11). As the value $k'' = 0$ corresponds to the assumption that the y values are free from error and $k'' = 1$ corresponds to the assumption that the x and y values have equal error, we take $k'' = e_y/e_x$.

Accordingly the required values of k are

$$k = -\frac{e_x}{e_y} \quad \text{when} \quad 0 \leqslant e_x/e_y \leqslant 1 \qquad . \quad . \quad . \quad (13)$$

and

$$\left.\begin{aligned} k &= \frac{e_y}{e_x} - 2 \quad \text{when} \quad 0 \leqslant e_y/e_x \leqslant 1 \\ &\text{or} \quad 1 \leqslant e_x/e_y \leqslant \infty \end{aligned}\right\} \qquad . \quad . \quad . \quad (14)$$

Expression (9) then becomes

$$\sum_{i=1}^{n} (A_i B_i)^{1-\frac{e_y-e_x}{e_y}} (A_i C_i)^{1+\frac{e_y-e_x}{e_y}} \quad \text{when} \quad 0 \leqslant e_x/e_y \leqslant 1$$

and

$$\sum_{i=1}^{n} (A_i B_i)^{1-\frac{e_y-e_x}{e_x}} (A_i C_i)^{1+\frac{e_y-e_x}{e_x}} \quad \text{when} \quad 0 \leqslant e_y/e_x \leqslant 1.$$

Both represent the sum of the weighted products of the distances of the points from the line measured in directions parallel to the x and y axes : weighting being in terms of the power of the distances. The individual weighting factors are :

$$[A_i C_i/A_i B_i]^{(e_y-e_x)/e_y} \quad \text{when} \quad 0 \leqslant e_x/e_y \leqslant 1$$

and

$$[A_i C_i/A_i B_i]^{(e_y-e_x)/e_x} \quad \text{when} \quad 0 \leqslant e_y/e_x \leqslant 1.$$

We note that for any non-zero value of r, the value of c corresponding to $e_x/e_y = e_0$ where $0 \leqslant e_0 \leqslant 1$ is the same as the value of $1/c$ corresponding to $e_y/e_x = e_0$. It follows from (8) that the relationship between c and e_x/e_y for $0 \leqslant e_x/e_y \leqslant 1$ and between $1/c$ and e_y/e_x for $0 \leqslant e_y/e_x \leqslant 1$ approximates to linearity. The departure from linearity decreases with increase in absolute magnitude of r.

The lines of best fit corresponding to different values of e_x/e_y and e_y/e_x when $r = 0{\cdot}5$ are shown in Fig. 3.

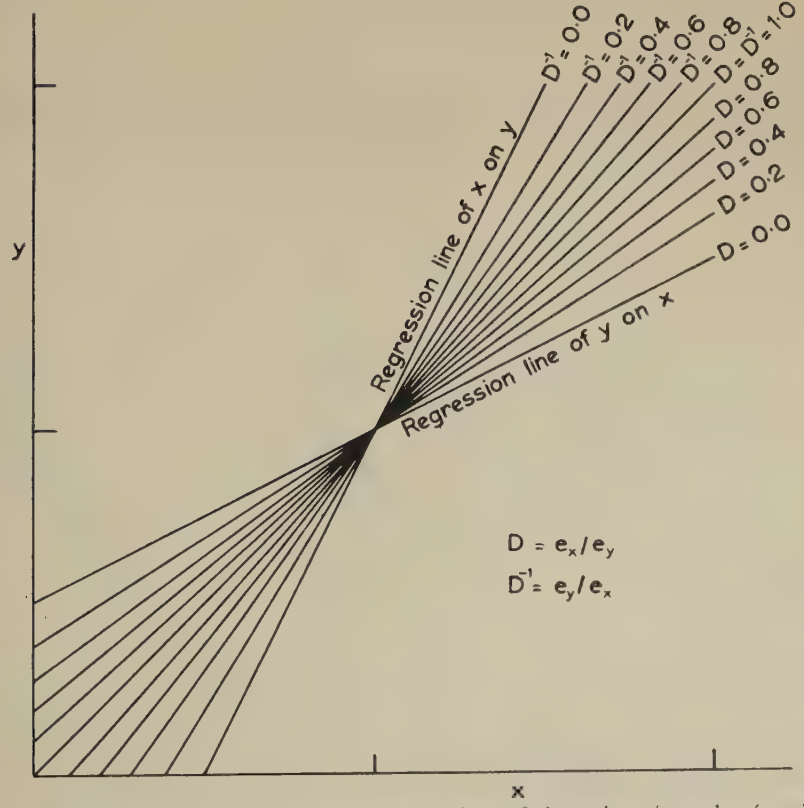

Figure 3. Lines of best fit corresponding to different values of the ratio e_x/e_y and e_y/e_x when $r = 0{\cdot}5$.

3. Remarks on practical application

The above analysis deals with values of the variates X and Y in terms of scaled values x and y respectively. But $s_x = \alpha$ and $s_y = \beta$ so that $\frac{x_i}{s_x} = \frac{X_i}{S_X}$ and $\frac{y_i}{s_y} = \frac{Y_i}{S_Y}$. Accordingly, the general Eq. (4) to the single line of best fit, and all special cases, hold without change of form for the original variates. Further, $e_x = e_X/S_X$ and $e_y = e_Y/S_Y$ where e_X and e_Y are the measures of the errors of the X and Y variates, as known from experience or other means. The type of measure used for expressing the uncertainty of the Y values should be the same as that used for the X values.

Examples

1. Variation of grass minimum temperature at Birr and Mullingar during June 1958. Values of the various constants derived from daily values are :

Birr $\quad \bar{X} = 44{\cdot}2°\text{F}, \quad S_X = 5{\cdot}7°\text{F}$

Mullingar $\quad \bar{Y} = 45{\cdot}2°\text{F}, \quad S_Y = 4{\cdot}9°\text{F}$

Coefficient of correlation, $r = 0{\cdot}85$.

Owing to identity of type of thermometer, exposure, method of observation, etc., at the two stations the error (e_X) at Birr is assumed to be the same as the error (e_Y) at Mullingar. Hence $\frac{e_X}{S_X} \Big/ \frac{e_Y}{S_Y} = \frac{S_Y}{S_X} = 0{\cdot}86$ and $k = -0{\cdot}86$. Accordingly $c = 0{\cdot}98$ and the line of best fit is

$$\frac{Y - 45{\cdot}2}{4{\cdot}9} = 0{\cdot}98\,\frac{X - 44{\cdot}2}{5{\cdot}7}$$

or

$$Y = 0{\cdot}84\,X + 7{\cdot}98$$

2. Variation of monthly potential evapotranspiration with monthly mean temperature near the ground at Valentia Observatory during 1957 and 1958. Mean temperature in the screen in the instrumental enclosure is used as an estimate of the mean temperature near the ground at the evapotranspirometer.

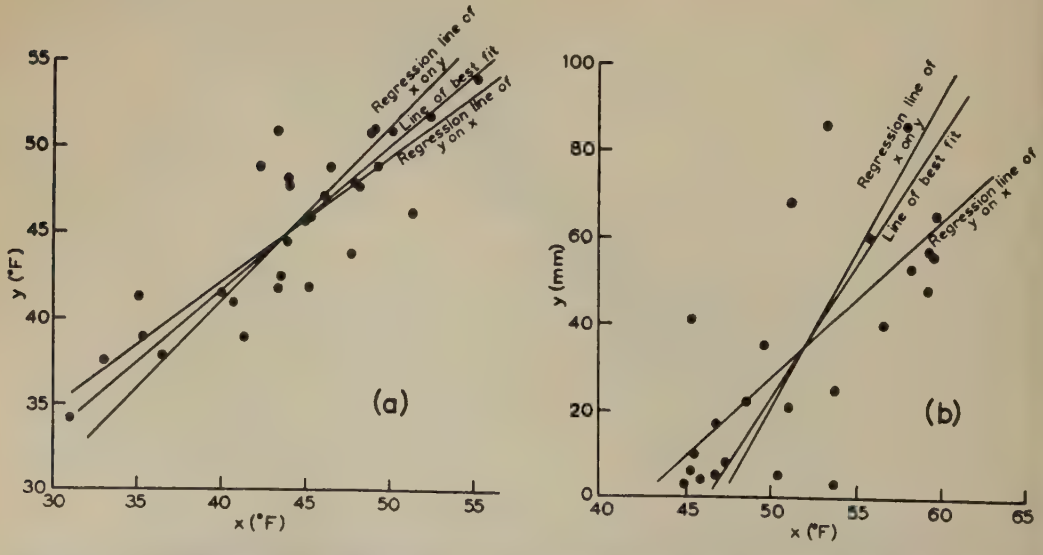

Figure 4. Scatter diagrams showing relation between (a) grass minimum temperatures at Birr and Mullingar during June 1958, and (b) temperature and potential evapotranspiration at Valentia during 1957 and 1958.

Values of the various constants are :

Temperature	$\bar{X} = 51{\cdot}9°\text{F}$,	$S_X = 5{\cdot}2°\text{F}$
Potential Evapotranspiration	$\bar{Y} = 34{\cdot}3$ mm,	$S_Y = 26{\cdot}7$ mm

Coefficient of correlation, $r = 0{\cdot}71$.

The standard deviation of the error in monthly mean ground temperature by assuming it to be given by monthly mean screen temperature is taken to be 2°F. The standard deviation of the error in the measured monthly potential evapotranspiration is taken to be 4 mm.

Then $\frac{e_X}{S_X} = 0{\cdot}38$ and $\frac{e_Y}{S_Y} = 0{\cdot}15$ so that $k = -1{\cdot}61$ and $c = 1{\cdot}20$. The line of best fit is

$$Y = 6{\cdot}16\,X - 285{\cdot}48$$

Scatter diagrams with regression lines and single line of best fit corresponding to the two examples are given in Fig. 4.

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Gumbel, E. J. 1954 'Statistical theory of extreme values and some practical applications.' *Appl. Math. Ser., U.S. Bur. Stand.*, Washington, No. 33.

551.524.7 : 551.571.7 : 551.576.11

Observations from aircraft of temperatures and humidities near stratocumulus clouds

By D. G. JAMES

(Read 21 October 1959. See *Q.J.*, **85**, p. 120)

Dr. R. S. Scorer : It is not correct to assume that fluctuations of vertical velocity as measured by an aircraft moving horizontally above the clouds indicate that there is a transport of water vapour or heat. In the very stable air above the cloud the fluctuations are probably due to the up-and-down motion produced by the convection currents in the cloud below, and are therefore wave motions which produce no vertical transport whatever.

The method of defining the level of the top of the cloud must be called in question. Was the cloud absolutely flat-topped ? If so, there cannot have been any substantial convection in it. If the cloud was not flat-topped, how was the level of the top defined ? From what we know of convection processes it can be concluded that there would be no appreciable transport of water vapour upwards at a level above that reached by the highest cumuliform clouds, but up to that height the convection could transport water vapour into the air which would, of course, lie above most of the clouds, but not all of the clouds.

Dr. James only gives it as an opinion that turbulent diffusion is the cause of the transport he infers to have taken place, but it appears equally possible to explain the events by supposing the right amount of subsidence or up-motion to explain the whole effect ! In such a case we must necessarily use what knowledge we have of the mechanisms which might produce an upward transport of vapour, and without knowing precisely where the highest cloud tops lay above the general cloud top, very little can be deduced except that if there were a transport of vapour to layers above the cloud tops it was almost certainly by the highest convection towers, and for these K-theory is inappropriate, at any rate so long as an attempt is made to associate K with the fluctuations of vertical velocity measured by the aeroplane.

Dr. W. Hitschfeld : Dr. James has been criticized by Dr. Scorer regarding the interpretation that had been placed on the accelerometer records. I cannot add to this specific argument; however, there is little doubt that mixing takes place in a layer of *some* thickness above cloud top. Whether such a layer is 100 ft or 300 ft, as Dr. James suggests, or 500 ft, in my view makes little difference to Dr. James's main thesis, since the heat and water-vapour capacities of such a layer are very small in comparison to that of the volume under discussion.

If one remains a little unconvinced about Dr. James's main conclusions regarding variations in K, this is probably due to the number of sweeping assumptions that had to be made, e.g., the fairly crucial cloud-albedo figure of 0·50, or the plausibility of the subsidence figures. Maybe Dr. James has shown us that in the detailed analysis of even so deceptively simple a system as he studied, the number of data required is generally greater than the number available.

Mr. F. H. Ludlam : I agree with Dr. Scorer that the accelerometer records show only that the airflow above the clouds was disturbed, and not that mixing motions were occurring. However, in my rather limited experience, the upper surface of a stratocumulus layer always shows cumuliform hummocks which rise 100-200 ft above the general level of the cloud tops and are clearly effecting a mixing between the cloudy and the clear air; sometimes, indeed, it can be seen from just above the tops that haze, characteristic of the lower well-mixed layer, extends a similar distance above the general cloud tops. I think it may be important to consider this convective mechanism of the mixing, for it might be that its efficiency is related to the thickness of the cloud layer in which it develops, with the result that over a considerable range of subsidence intensities the cloud thickness becomes adjusted to a particular rather stable value. I suspect that a sheet of cloud disappears over a large area when the rate of subsidence becomes excessive, as may happen when a diurnal component has been added to it, rather than because of a change in the efficiency of the mixing. Dr. James is to be congratulated on making the first careful study of the equilibrium of this kind of cloud.

Dr. R. C. Sutcliffe, F.R.S. : It is possible, as Dr. Scorer has implied, that the ' turbulence ' experienced by the aircraft above the cloud top was a wave-like streamline flow implying no vertical flux of moisture, although I think this is improbable. However, this point is not very important. More interesting is the basic model of the inversion as a dynamical balance between turbulence spreading upwards and general subsidence. The inversion is not a down-slide surface but is essentially a turbulence inversion. This is not a new concept; I think it was C. K. M. Douglas who first explained this mechanism to me many years ago, but it is good to have this new evidence. One has in mind, also, recent evidence that frontal inversions are not necessarily up-slide surfaces, and also that the tropopause may typically be moving upwards relative to a subsiding atmosphere. These seem to be related phenomena suggesting that sharp discontinuities of lapse-rate should not generally be treated as air-mass boundaries.

Mr. B. C. V. Oddie : There can surely be no doubt that mixing does take place between the cloud and the dry air above. If it were not so, the regions of descending air would have the same water content and therefore the same optical density, as the ascending parts. In fact, the descending parts are always much thinner, and are frequently quite clear, and this can only be because mixing has taken place with dry air at the cloud top.

Dr. C. D. Walshaw : I understand that on some occasions Dr. Brewer finds a small rise in the ground-level ozone concentration in the early hours of the morning. This could be interpreted as a consequence of the increase in turbulence postulated by Dr. James; it would be interesting to know whether this ozone behaviour is associated with stratocumulus cover.

Mr. W. G. Durbin : I think it would be better to investigate the problem of the nocturnal dissipation of stratocumulus at night, and since the present radiosonde rose too rapidly for accurate temperature and humidity measurements to be made, a series of ascents made with balloons rising at 100 or 200 ft per minute to cover the period of dispersal of the cloud might well yield valuable information on the physical processes involved. Until these processes are more fully understood it is unlikely that the standard of forecasting this phenomenon will improve.

Dr. R. C. Rainey (*communicated*) : Dr. James's records of marked temperature-inversion, hydrolapse, and decreasing gustiness above these clouds can be paralleled from some incidental observations made near Nairobi (5,500 ft above sea-level) in July 1951, on the persistent stratocumulus sheet which is very frequently to be found in this area at this time of year. Thus at 0725-0940 hr on 6 July, with cloud-base at 6,700 ft and cloud-tops just below 7,700 ft, air temperatures were 48°F at 7,200 ft and 50° at 7,600 ft, with a corresponding fall in specific humidity from 9 to 7 g/kg; and only a single gust exceeded the thresholds of 0·1 g above and below the 1 g datum during a 7 min descent from 9,750 to 8,250 ft, whereas during 15 min further descent, from 8,240 to 5,600 ft, both these thresholds were passed 13 times, including 5 gusts exceeding + 0·2 g, of which one exceeded + 0·3 g.

Concerning Dr. Scorer's misgivings as to whether such accelerometer records must necessarily be evidence of turbulence, are not these doubts to some extent dispelled when the recorded gustiness decreases with height as abruptly as, for example, in Dr. James's Fig. 9 ?

Finally, does not the persistence of such cloud imply continuing supplies not only of dry air from above, by subsidence (which, alone, would presumably disperse it), but also of humid air from below ?

Dr. D. G. James (*in reply*) : To reply to Dr. Scorer's first point I must draw his attention to portions of the accelerometer records shown in Fig. 9 of my paper. An evaluation of a turbulence index (the sum of areas per unit time between the accelerometer trace and a zero line) shows little change with height from below cloud base to 200 ft above cloud top and abruptly falls almost to zero at 300 ft above cloud top. I think this is inconsistent with the postulation that a wave motion in the inversion is solely responsible for the records obtained.

I agree with Dr. Scorer that the cloud top was not perfectly flat, but the aircraft flying at 100 ft above this level was quite clear of cloud, and photographs taken during the flight showed that variations in cloud top were of the order of tens of feet only. The indicated turbulence was evident considerably higher than this. At night when the cooling of the cloud is very much greater, these variations may well be considerably larger, but I have no observations to this effect.

The solution of the problem which I present is not unique. As Dr. Scorer and Mr. Ludlam point out, an increase in subsidence by night could be sufficient to disperse the cloud. This would be accompanied by a decrease in height of the inversion base which very likely does occur occasionally. However the evidence of Neiburger, mentioned in my paper, shows that the converse is more probable, and my solution fits these observations.

I agree with Dr. Hitschfeld that sweeping assumptions have been made in the argument, in particular the assumption of 0·5 for the albedo of the cloud. As frequently happens in physical

meteorology, I am placing emphasis on the small differences between large quantities. A decrease of 0·1 in the albedo, for example, would mean an increase in K by a factor of 2 for the same heat balance to be maintained.

I am most interested to hear of Dr. Rainey's experiences with stratocumulus cloud at Nairobi, his accelerometer records agreeing well with those shown in Fig. 9. His final point concerns the availability of humid air to allow the cloud to persist. I suggest that the reservoir of water vapour for the cloud is the air below. The origin of the turbulence is near cloud top where the long-wave cooling is greatest, the consequent cooling of the cloud causing an upward flow of moisture from the air below.

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Radar echoes from atmospheric inhomogeneities

By R. F. JONES

(Read 21 October 1959. See *Q.J.*, **84**, p. 437)

Mr. W. G. Harper : I find myself placed in a rather peculiar position in that I have been using slides similar to those shown by Mr. Jones as typical examples of the migration movements of birds. There is a growing body of evidence that the majority of these ' unexplained ' effects on our radars are due to feeding flights, roosting and migration movements. In fact, radar displays are currently being used to learn more about the migration of birds (e.g., Lack, D., ' Migration across the North Sea studied by radar,' *Ibis*, **101**, 1959, pp. 209-234).

Despite this I am convinced that a residue of unexplained echoes remains, and for these a meteorological explanation is a strong possibility. I am unhappy however about Mr. Jones's atmospheric inhomogeneities caused by side lobes (see *Q.J.*, **85**, p. 435). If an echo were caused by side-lobe energy, and were present from sweep to sweep, then the whole side-lobe pattern must be displayed at that range, not an isolated part of it as Mr. Jones seems to imply. Careful observation does not support this.

Prof. P. A. Sheppard : Since birds are a proven source of ' angels ' on many cocasions, can Mr. Jones say categorically that some at least of his cases of ' angels ' were not accompanied by birds in sufficient density to account of the echoes.

If there are such cases, under what meteorological conditions – subsidence inversions apart – are they most likely to occur ?

Dr. D. Atlas : In response to Professor Sheppard's question regarding the existence of any radar-' angel ' observations in which the absence of birds, insects, or other particulate matter could be definitely confirmed, I should like to bring attention to our 1·25 cm observations of the sea breeze which were first reported in the *Proceedings of the Conference on Radio Meteorology* (University of Texas, November 1953). In that case, the ' angels ' were closely correlated with the refractive index inhomogeneities associated with the onset of the sea breeze. The use of a narrow (0·37 degree) beam directed toward the sea at an elevation of only one degree from a point ½ mile inland, provided a unique opportunity for direct visual observation of the radar pulse volume by an observer at the shore with and without the aid of binoculars. No targets could be seen visually throughout the 2-hr periods of the sea-breeze echoes, except for a few birds which were present in roughly the same number as prior to the onset of the ' angels,' and accounted for the few dot ' angels.'

Mr. Jones's attempt at explaining the occurrence of ' angel ' echoes is an ingenious one and, I feel, it must account for some of the ' angels ' which have been observed. However, I have several objections to the theory as a general explanation of stratiform ' angel ' echoes. My first two objections are based on observational experience. In the February 1959 issue of the *Journal of Meteorology*, I reported the observation of layer-type ' angels ' generally similar to those reported by Jones and Harper. In the few cases reported, and in others, the heights of the echo layers, which ranged from 2,000 to 11,000 ft, agreed closely with definite layers of moisture minima or subsidence inversions shown in the radio soundings. These are the layers in which moist air

rising from the surface will show strong moisture and index contrast with the dry environment and provide favourable conditions for direct back-scatter. Although I realize the great difficulty in explaining the echo magnitudes theoretically on the basis of direct back-scatter, the strong correlation of ' angel ' layers to meteorological layers cannot be avoided. This would not be expected if Mr. Jones's theory were applicable.

Secondly, the observations were made on the narrow Cape Cod peninsula which extends out into the Atlantic Ocean from the Massachusetts coast. In a number of cases, the ' angel ' layers extended well out to sea, considerably beyond the range of any possible ground targets. Of course, there could have been back-scatter from a rough sea, but this is unlikely since there was a notable preference for the angels to occur under anticyclonic conditions and light winds.

My third objection has already been mentioned by Mr. Harper, i.e., a ground target detected in the manner proposed is likely to appear as a vertical column on the RHI. This could not account for the thin well-defined ' angel ' layers to which I have referred, and neither can Mr. Harper's birds account for such ' angel ' layers. In short, the evidence still indicates that some ' angels ' do in fact occur by direct back-scatter from sharp atmosphereic inhomogeneities.

Dr. J. A. Saxton : There is no doubt that radar echoes at centimetric wavelengths are obtained from birds, and that some ' angel ' phenomena are certainly attributable to this cause. On the other hand, I think it would be quite wrong to suggest that all echoes of this type must be due to birds and that the mechanism of ' angel ' production described by Mr. Jones is not a possible one. Experiments have been in progress at the D.S.I.R. Radio Research Station for several years in which a high-power 10 cm-wavelength radar is used to make vertical incidence soundings of the lower troposphere. This investigation has revealed that clearly defined scattering or reflecting layers of limited horizontal extent exist for much of the time. The pulse-length of the radar (0·1 microsec) does not permit the resolution of layers thinner than some 30 m, but it is evident that layers of this thickness, and very probably less, occur not infrequently. This atmospheric structure is amply borne out by microwave refractometer measurements carried out in America and France, and also recently by the Radio Research Station in collaboration with the Meteorological Research Flight. It is clear that the variation of refractive index within these layers is such as to give relatively strong reflection of obliquely incident 10 cm waves, and I see no reason therefore why some ' angel ' echoes at least should not arise in the manner proposed by Mr. Jones.

Dr. R. C. Rainey (*communicated*) : Birds are, of course, not the only biological ' angels.' Radar sightings of desert locust swarms, for example, have been recorded in the Persian Gulf (*Nature*, **175**, p. 77, 1955); in Morocco, from Agadir (C. Rungs, unpublished observation); and in the Sudan, from Khartoum (U. Rath, *in litt.*).

Mr. D. C. Thomas (*communicated*) : During an investigation into atmospheric motions with the aid of a radar at the Imperial College Field Station, Ascot, several detailed observations of ' angel ' echoes were obtained. These differ from the results shown by Mr. Jones since the echoes were studied singly or in small groups.

The radar used (a 10 cm G.L. 3, Mk. VII) was designed primarily for lock-follow on an echo received from a single reflector, ignoring all echoes received from other reflectors unless they are very close to the required reflector. The P.P.I. display is much inferior to the one in the East Hill radar shown by Mr. Jones and is of little use for observing ' angel ' echoes. The A-scope display incorporates a high-resolution trace on which it is possible to distinguish between echoes from reflectors on the same bearing and elevation differing in range by 50 m.

On some days ' angels ' could be found very easily, and the radar maintained in lock-follow on each one until the range became too great to give a satisfactory signal. Records were obtained for some 30 cases in which lock-follow lasted between 5 and 15 min. In most cases the ' angel ' appeared to move at between 20 and 40 kt relative to the wind, maintaining constant height and direction during the time it was followed, an exception to this being a case when the angel rose from 900 m to 1,900 m in 11 min. In some cases the ' angel ' behaved as a point-source reflector and in others it was possible to distinguish separate echoes on the A-scope display. The variation of echo strength with range indicated that the angel maintained a constant-echoing area during the time it was followed, but in one case the echo strength varied markedly at approximately 4 c/s.

In three of the cases it was possible to see birds in the telescope which is attached to the aerial assembly, and in each of these cases the apparent motion of the birds relative to the graticule of the telescope was characteristic of the radar hunting on a reflector when in lock-follow. It was never possible to see birds at the ranges concerned without the aid of the telescope, nor was it possible to locate a bird with a pilot-balloon theodolite using bearing and elevation supplied by

the radar. A goose subtends an angle of $\frac{1}{3}$ min of arc at a range of 5 mi so it is not surprising that birds were never visible to the naked eye. I think that all the results are consistent with the assumption that the echoes were from birds, and on several occasions the radar could have been tracking geese or swans. There is a daily migration of water birds to and from the reservoirs at Staines and it is almost certain that some of these pass near Ascot.

Dr. Saxton's record is very interesting. I doubt if birds would all fly at the one height but would like to suggest the possibility of particulate matter being stratified in certain weather conditions.

Mr. R. F. Jones (*in reply*) : The fact, as Mr. Harper says, that radar displays are currently being used to learn more about the migration of birds is not, of course, evidence of the correctness or otherwise of the hypothesis that displays of the type I discussed are caused by migrating birds. It only shows how desirable it is that the question should be resolved. The great majority of 'angel' echoes are received at low angles of elevation of the main lobe; at short ranges where one might expect to see more of the side-lobe pattern the display is complicated by permanent echoes. I doubt, therefore, whether one would expect to see a clear side-lobe pattern as Mr. Harper and Dr. Atlas suggest.

Dr. Atlas, in comparing the layer-type echoes which he observed with those which Mr. Harper and I have discussed, overlooks an important point. There is no possible doubt that our echoes moved with an apparent speed in excess of the wind speed and cannot therefore have arisen from direct back-scatter. This is the crucial point in the whole discussion and if Dr. Atlas's echoes did move with the wind speed then he was certainly seeing a different type of echo, for which direct back-scattering may provide an explanation. Not unexpectedly I found no correlation between 'angel' layers and meteorological layers on two occasions when detailed readings in the vertical, obtained from a tethered balloon within the general area of the echoes, were available. I do not know how calm the sea may be and yet still give some back-scatter to be observed at the very small glancing angles which I would require (less than $\frac{1}{2}$ degree) but even the disturbance in light winds might be enough.

Dr. Saxton's observations are most interesting and give more confidence to those who believe in the existence of reflecting layers in the atmosphere.

I do not doubt that Mr. Thomas was following birds with his radar, for there can be no doubt that birds and locust swarms are detectable with high-power radars, but with him I doubt very much whether they would explain Dr. Saxton's records. The explanation he proposes in terms of particulate matter is unattractive since the diameter of such suspended particles must be very small and even the high concentrations of droplets associated with clouds are usually not detected with the type of equipment Dr. Saxton used.

In reply to Professor Sheppard, I do not accept that because flocks of birds have been shown, not surprisingly, to give an echo, they are a 'proven source' of the type of 'angels' I have discussed. It is, however, obviously impossible to state categorically what the distribution of birds was on any occasion over a circular area of 30-50 mi radius, especially as the more striking displays were associated with the dusk period. It *is* possible to determine a bird distribution which would account for a particular echo display, but even the ornithologists are unable to give independent proof of the presence of birds in the required numbers. Instead, apparently, they accept the radar evidence as proof of the existence of large numbers, even up to 3 million in 2 hours on a 60 mi front.

The most favoured meteorological conditions for extensive 'angel' displays were clear skies in the period around sunset with light winds and rapidly falling surface temperature. On these occasions 'angel' activity increased around dusk simultaneously over the whole display, not by the influx of echoes from a single direction as one might expect with migrating birds (of course, the birds might have been roosting evenly in the area around, waiting for darkness). In a recent letter to *Nature* (**184**, p. 1059) Edwards and Haughton have shown that the equivalent echoing area of a bird head-on or tail-on is less than 1/10 of that when viewed broadside-on but there has been no evidence on the radar displays of an elliptical-echo distribution as the bird hypothesis would require.

Proceedings at meetings of the Society

Summer Meeting, Durham, 17-19 September 1959

The sixth two-day Summer Meeting of the Society was held in the Department of Physics, The Durham Colleges, University of Durham, by permission of the Warden, Sir James Duff, M.A., M.Ed., LL.D., D.C.L., and the Head of the Department, Professor G. D. Rochester, M.Sc., Ph.D., F.R.S., F.I.P.

The meeting was formally opened on the evening of 17 September by Professor and Mrs. G. D. Rochester, and Dr. E. M. Williamson, Principal of St. Mary's College.

Professor G. D. Rochester took the Chair at the first Symposium on ' Atmospheric electricity.' Speakers included Mr. J. E. Maund, whose paper with Dr. J. A. Chalmers on ' Point discharge currents from natural and artificial points ' was presented, followed by Mr. J. Adamson on ' Compensation for potential gradient changes in measurement of air-earth current.' Dr. T. W. Wormell presented a paper by Professor Dr. H. Israël on ' Atmospheric electrical agitation '; Mr. C. J. Adkins presented his paper on ' The small ion concentration and space charge in the lower atmosphere using field mills.'

Professor P. A. Sheppard, President of the Society took the Chair at the second day's symposium on ' Climatic change.' Contributors included Dr. A. E. M. Nairn, Professor Gordon Manley and Mr. H. H. Lamb : the session concluded with a Discussion.

An enjoyable tour by coach to the Roman Wall took place on the Friday afternoon, and was attended by the President and some thirty members and guests, and the guide was Mr. R. P. Wright, F.S.A., M.A. On Saturday afternoon a tour was made of Durham Castle and Cathedral.

Council wish to record their sincere gratitude and thanks to Dr. A. G. Forsdyke who acted as co-ordinator and to Dr. J. A. Chalmers and his colleagues on the Local Committee who had helped to make the meeting a success scientifically and socially.

(A full account of the meeting appeared in the January issue of *Weather*).

Ordinary Meeting, 21 October 1959

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Blackall, Rodney Martin, 78 Crescent Road, Leigh-on-Sea, Essex (*transfer from Student Membership*).

Horne, Edgar, F.P.S., 22 Woodlands Drive, Brooklands, Sale, Cheshire.

Hull, Oscar Michael, 23 Jackson's Lane, Highgate, London, N.6.

Jeffries, Brian Lewis Arthur, Lamorna, 83 Beechwood Avenue, St. Albans, Herts.

Lamond, Malcolm Hugh, B.Sc., 4 Third Avenue, Applecross, Western Australia.

Pennant, Arthur Dermot Humbertson, B.A., Nantlys, St. Asaph, Flint.

Prabhakar, Dilbagh Rai Lakhmidass, M.A., c/o Hydraulic Branch, P.O. Box 30043, Nairobi, E. Africa.

Qureshi, Mohammed Yousuf, B.Sc., 102 Sydney Street, London, S.W.3. (*transfer from Student Membership*).

Smith, Cecil Everton Aurelius, B.Sc., W.I.M.S., Meteorological Office, Piarco Airport, Trinidad, West Indies.

Taylor, Bernard Horace Theodore, 26 Dale Hey, Hooton, Wirral, Cheshire (*reinstatement; elected 1945; resigned 1955*).

Worsnop, Alan Robert Shaw, Park Crest, 76 Holly Bank Road, Bradford 1, Yorks. (*reinstatement; elected 1945; resigned 1954*).

Canadian Branch

Fellow

Barklie, Robert Henry Douglas, M.Sc., A.R.I.C., Apt. 5, 3626 Mountain Street, Montreal.

The following candidates were then declared to have been elected that day by the council :

FELLOWS

BAKER, Helen Muriel, St. Helens, 9 Forest View, Chingford, London, E.4.
BERRY, Keith, B.A., Hazeldene, 113 Hough Lane, Wombwell, Nr. Barnsley, Yorks.
COX, Arnold Elmslie, The Haven, Wootton Rivers, Nr. Marlborough, Wilts.
CROSS, Donald Alfred Edgar, B.A., F.R.G.S., 54 Ducklington Lane, Witney, Oxon.
DAVIES, Donald Lewis, B.A., Maitland, North Avenue, Llandrindod Wells, Radnor.
DIXON, Charles, B.Sc., 23 Johnston Avenue, Dundee, Scotland.
ELLIOTT, James Alderson, 96 Montpellier Terrace, Cheltenham, Glos.
FENN, Flt. Lt. Michael George, R.A.F., 53 Claremont Road, Bishopston, Bristol 7, Somerset.
GUNTON, Henry Edward, 13 Ash Tree Road, New Costessey, Norwich, Norfolk.
HUTCHINS, Flt. Lt. Ian Albert, A.F.C., c/o 75 Sqn. R.N.Z.A.F., R.A.F. Tengah, Singapore 24.
KINGTON, John Alfred, B.Sc., 2 Greenwood Drive, Kingswood, Watford, Herts.
LIMBRICK, Sgt. Gordon, Sergeants' Mess, R.A.F. Aldergrove, Crumlin, Co. Antrim, N. Ireland.
MADDEN, Flt. Lt. Francis Anthony Patrick, 127 Stanley Road, Kirkdale, Liverpool, Lancs.
PRICE, Trefor, Bryngwyn, Graigwen Road, Pontypridd, Glamorgan (*reinstatement; elected 1947, resigned 1951*).
RUSHFORTH, John Mackinnon, B.Sc., Ph.D., Mathematics Department, Queen's College, Dundee, Scotland.
WOODACRE, Thomas Sidney, 63 Sylvan Avenue, Bitterne, Southampton.

STUDENT MEMBERS

EDWARDS, Michael George, 27 Bridge Cross Road, Chase Terrace, Walsall, Staffs.
HIRST, Terence Lawson, Bardsey, 37 Pine Walk, Woodmansterne, Banstead, Surrey.
LEWIS, John Barry, 8 High Street, Much Wenlock, Shropshire.
SPENCE, Jean Melville, Avalon, Kingston, Nr. Lewes, Sussex.
WILSON, Donald James, Springfield Villa, Dacre Banks, Harrogate, Yorks.

The following papers were read and discussed :

1. ' Observations from aircraft of temperatures and humidities near stratocumulus clouds ' by D. G. James (*Q.J.*, **85**, p. 120).

2. ' Radar echoes from atmospheric inhomogeneities ' by R. F. Jones (*Q.J.*, **84**, p. 437).

Evening Discourse, 4 November 1959

The first of three evening Discourses planned for the 1959-1960 Session took place in the Society's Rooms : Mr. H. Charnock spoke on ' Ocean currents.'
(A full account of the meeting appeared in January issue of *Weather*).

Ordinary Meeting, 18 November 1959

The following papers were read and discussed :

1. ' Exchange of heat, moisture and momentum between *Hurricane Ella* (1958) and its environment ' by M. Ghangopadhyaya and H. Riehl (*Q.J.*, **85**, p. 278). Presented by Professor H. Riehl.

2. ' Mean meridional circulation in the atmosphere ' by G. B. Tucker (*Q.J.*, **85**, p. 209).

Popular Lectures

30 November 1959 : Mr. A. L. Maidens took the Chair for the first of the Society's series of Popular Lectures for 1959-1960. The meeting was adressed by Mr. J. MacDowall, O.B.E., who gave an account of the meteorological work at Halley Bay during the International Geophysical Year, his subject being entitled ' Meteorology in Antarctica.' The Popular Lectures are held annually for the benefit of sixth-form pupils in the London area, The lecture was delivered in the Conference Hall of the London County Council by invitation of the Chief Education Officer of the L.C.C.

7 December 1959 : Dr. J. M. Stagg, C.B., O.B.E., M.A., D.Sc., President of the Society, was in the Chair when Mr. J. MacDowall repeated his lecture, which was again held in the Conference Hall of the London County Council and which was attended by pupils from schools in Buckinghamshire, Essex, Kent, Hertfordshire and Surrey, together with a number of pupils from independent schools.

The total attendance at the two lectures in the Conference Hall was about 850.

Evening Lectures on Meteorology, 9, 10 and 11 December 1959

Professor P. A. Sheppard, Hon. A.R.C.S., B.Sc., F.Inst.P., presented the first three of the new series of Evening Lectures held in the Society's rooms before a specially invited, limited number of sixth-form pupils and their science teachers. These lectures are intended for pupils who will read mathematics and/or physics at university.

The title of the lecture was ' Heat phenomena in the atmosphere : the atmosphere as a stage for the demonstration of the laws of heat.' Its aim was to show pupils how the physics and mathematics with which they are familiar can be applied in meteorology.

The invitation to attend was accepted by some 150 pupils and their teachers.

(A full account of the Evening Lectures will appear in the February issue of *Weather*).

Ordinary Meeting, 16 December 1959

The following candidates were declared to have been elected that day by the Council :

FELLOWS

BETHELL, John Pennington, B.Sc., M.Sc., 34 Mather Road, Birkenhead, Cheshire.
CHALMERS, William, Manor Farm, Kilham, Driffield, Yorks.
DALTON, Arthur Brian, Beecholme, Derby Road, Nottingham (*transfer from Student Membership*).
DE SILVA, Mataramba Bala-aratchige Gunatunge, B.Sc. (Ceylon), Department of Meteorology, Bullers Road, Colombo 7, Ceylon.
FERNANDES, Kenneth Julian, 51 Chuim, Khar, Bombay 52, India.
FORD, Maurice Walter, April Cottage, 7 Back Street, Abbotsbury, Dorset.
GLENNON, Gerard Joseph, Meteorological Office, R.A.F., Geilenkirchen, B.F.P.O. 40.
GOLDSMITH, Philip, B.A., 18 Fitzcount Way, Wallingford, Berks.
HARVEY, John Gordon, B.Sc., Fisheries Laboratory, Lowestoft, Suffolk.
HEATHCOTE, John, B.Sc., Dip. Ed., Prince of Wales School, P.O. Box 30047, Nairobi, Kenya.
HINDLEY, David Reed, Officers' Mess, R.A.F., Geilenkirchen, B.F.P.O. 40.
HRYNKIEWICZMOCZULSKI, Marius Boleslaw, 17 Ambleside Avenue, Streatham, London, S.W.16.

Jackson, Charles Ian, B.A., M.Sc., Department of Geography, London School of Economics and Political Science, Houghton Street, Aldwych, London, W.C.2. (*transfer from Student Membership*).
Karunatilleke, Don Premasiri Weerakoon, B.Sc., 4 Lily Avenue, Jayanthipura, Talangama, Ceylon.
Levett, Richard Edward Keith, M.A., 122 Regent Street, London, W.1.
Muffatti, Arthur Henry John, B.Sc., 34 Henley Road, Mount Pleasant, Western Australia.
Paley, James, B.A., Nicol House, Lazonby, Penrith, Cumberland.
Patrick, Godwin Olu, B.Sc., c/o Department of Meteorology, Massachusetts Institute of Technology, Cambridge, Massachusettes, U.S.A.
Richards, John Morgan, B.Sc., Flat A, 37 Cromwell Road, London, S.W.7.
Rickers, Denis, 97 Ruabon Road, Wrexham, Denbigh.
Shore, William Robert, P.O. Box 440, Mombasa, Kenya.
Smith, Stanley Desmond, B.Sc., Ph.D., 21 Simon's Lane, Wokingham, Berks.
Subrahmanyam, Varanasi Parvati, B.Sc., M.Sc., M.A., D.Sc., Department of Geophysics, Andhra University, Waltair, South India.
Tyldesley, John Birtwell, B.A., Meteorological Office, Huntingdon Road, Cambridge.

Student Members

Cox, David Lloyd, 3 Bedale Avenue, Scarborough, Yorks.
Howell, Jacqueline Ann, 54 Hempstead Road, Watford, Herts.
Morrissey, Edward Graeme, 53 Halkett Place, St. Helier, Jersey, C.I.

Foreign Members

Mané, Uri, Israel Meteorological Service, P.O. Box 7036, Tel-Aviv, Israel.
Ohtake, Takeshi, M.Sc., c/o Geophysical Institute, Faculty of Science, Tohoku University, Katahiracho, Sendai, Japan.

Canadian Branch

Fellow

Catling, Ronald George, B.A., 1285 Wellington Avenue, Winnipeg, Manitoba.
McBride, John Harold, B.Sc., 7555 Berri Street, Montreal 10, Quebec.
Gates, Archibald Dunham, B.Sc., 20 Lawnsdale Drive, Dartmouth, Nova Scotia.

The following papers were read and discussed :

1. ' Patterns of hailstorms in Alberta ' by R. H. Douglas and W. Hitschfeld (*Q.J.*, **85**, p. 105). Presented by Dr. W. Hitschfeld.

2. ' Seasonal and other temperature changes in the Antarctic atmosphere ' by H. Wexler (*Q.J.*, **85**, p. 196). Presented by Mr. J. MacDowall.

Reviews

General meteorology. By H. R. Byers. New York (McGraw-Hill), 3rd Edition, 1959. Pp. x, 540; 223 Figs.; 10 Tables. 74s.

The second edition of this well-known book was published in 1944, and it has remained among meteorologists as a popular book on account of its fairly wide scope and because of the compact form in which the information is given. The new edition introduces fresh material, but half of the chapters are practically unchanged, and there seems no reason why it should be otherwise. In the first line of the book Professor Byers makes acknowledgment to progress in other sciences by replacing the opening words " The sun " by " That great nuclear reactor, the sun " but thereafter there is little change in the first nine chapters beyond occasional revision to bring the text up to date. The publishers claim that ' practically all the illustrations are new,' but this is manifestly untrue. However, as Professor Byers says in the introduction, his book is concerned more than ever with fundamentals, and fundamentals are not easily outdated.

As before, some 200 pages are devoted to subjects such as radiation laws and the heat balance, observations and the instruments with which they are made, thermodynamics and the equations of motion. Chapter 10, on fluid motion, is a concise introduction to conceptions which have taken a prominent part in meteorological theory since the war, such as the conservation of vorticity, but little attempt is made here to show the application of these tools, and the computation of the speed of atmospheric waves, for example, is postponed to a later chapter.

The chapters on general circulation and the vertical structure of pressure systems contain a good deal that was not known when the earlier edition was written, and they now include momentum exchange on a circumpolar scale and discuss the role of the jet stream. The exposition of the development and life cycle of cyclones is on traditional lines and is not very successful in maintaining the three-dimensional approach. Certain explanations are not convincing. Ageostrophic frontal movement is said to be accounted for by frictional retardation, and the words ' convergence ' and ' divergence ' are noticeably absent.

Air masses are dealt with on a largely physical basis, regional statistics being omitted. (Type mAw is restored to the Bergeron classification. The student was invited to account for its omission in the previous edition, but apparently failed to do so !) The chapter on synoptic analysis is an interesting and practical outline of the broad three-dimensional exploration of the atmosphere, but forecasting is largely confined to ' Quantitative approaches to forecasting.' The author regards a general textbook as unsuitable for explaining the complexities of day-to-day synoptic work. ' Quantitative approaches ' deals solely with the production of a prognostic chart, and includes the movement of upper waves and the essentials of numerical forecasting. No attempt is made to discuss graphical methods but, somewhat surprisingly, the Petterssen method of computing displacements is described in detail as in the earlier text (and is illustrated by the same synoptic situation).

The author does not deal with the forecasting of such elements as rain and temperature, but the treatment of the processes of condensation and precipitation reflects the advances of recent years, as does the chapter on tropical cyclones, which embodies the work of Riehl and Palmén. Stagnation in the study of fog is confirmed by a chapter which is little changed apart from the introduction of ' smog,' though reference is made to statistical forecasting methods.

Professor Byers recognizes that it is increasingly difficult for a single book to achieve completeness over a range of subjects, and *General meteorology* is not intended to be self-sufficient at any level. Nevertheless, it may be questioned whether an introductory text can succeed in impressing fundamentals without explaining their application as widely as possible. However, this book is full of good material and is devoid of superfluity, and it is very reasonable in its demands on the reader's scientific knowledge.

C.J.B.

Weather. By R. S. Scorer. London (Phoenix House), 1959, Pp. 63; 36 Plates, 26 Diagrams. 9s. 6d.

This stimulating little book from the pen of Dr. Scorer forms one of the *Progress of Science* series, edited by Nigel Calder. The aim of the series is to satisfy the intelligent young student who wishes to find out something about a scientific subject beyond what is to be found in a conventional elementary text; in particular, one who wants to hear also about the progress of recent research in the subject. Dr. Scorer has fulfilled this aim admirably for meteorology.

The book comprises eleven short chapters including a brief final one on careers in meteorology. The style is lively and entertaining and the author's enthusiasm for his subject comes through well and imparts to meteorology an exciting flavour which should arouse the interest of potential meteorologists amongst the book's readers. This is achieved without superficial dramatisation and the book remains instructive, informative and authoritative throughout.

Analogies are freely used where they are helpful and so we have a whirling skater to illustrate the conservation of angular momentum, rather surprisingly a cyclist riding up and down hills to assist the discussion of the relation between wind and pressure gradient, and, of course, the bath plug ! Some instructive examples of real applications of meteorology are also used, e.g., the siting of French villages in the region subject to Mistral winds, and the siting of factories to minimize the nuisance of smoke pollution, etc.

The line diagrams are excellent, as also are the generous quota of cloud photographs with which the book is illustrated. Indeed, the treatment of clouds and cloud physics is perhaps the best part of the book, both as regards the text and the shrewd interpretations accompanying the photographs.

The remarks on careers are brief but to the point, although it is a little surprising to read here that research is mainly the province of the universities.

The student will find this a valuable introduction to the subject. The professional meteorologist will find it provides an enjoyable evening's reading and he will be very unusual if he does not also learn something new.

G.A.C.

Solar-Terrestrische Beziehungen in Meteorologie und Biologie. By H. Berg. Leipsig (Akad. Verlag., Geest & Portig), 1957, Pp. viii, 172; 77 Figs.; Tables. DM.23.

Solar-terrestial relationships form a field of research in which, broadly speaking, the cogency of the conclusions is not commensurate with the effort devoted to the investigation. The reader often works through a long, involved and controversial paper only to find that the author has failed to prove his case. Professor Berg sets out to give a systematic summary of the literature, covering about 200 individual research papers, and commenting on the validity of the results. He deals first with the measurements of the solar variables themselves, then with their associations with meteorological variables, and finally with their relations to some biological and medical statistics. The results are generally illustrated by the diagrams given by the original authors.

Since many authors do not publish their basic data, and since the initial processing of the data can in some circumstances generate oscillations of the kind found in many of the papers, it is not to be expected that a survey of this kind, which deals only with the end products, should always enable us to distinguish the genuine results from the rest. However, the book is to be recommended as a worthwhile attempt to bring a large body of work into proper perspective. It should help the serious student to decide which results will repay further study, either by reference to the original literature or by further investigation based on the same or additional basic data.

The book is well produced and the style easier, to an English-speaking reader, than that of many German authors.

J.M.C.

Klima und Bioklima von Wien. III Teil. By F. Steinhauser, O. Eckel and F. Sauberer. Vienna (Austrian Met. Soc.), 1959. Pp. 136; 37 Figs.; 36 Tables; 48 Austrian schillings.

This volume completes the study of the climate of Vienna put in hand on behalf of the city council some years ago to provide comprehensive information for planning and development. The earlier parts, which appeared in 1955 and 1957, were reviewed in the *Quarterly Journal*, **82**, 351, pp. 115-6 and **83**, 358, pp. 563-4. Part III is concerned with the detailed distribution of precipitation, and especially snow, of wind and of atmospheric pollution in the city and its environs. Pollution of all types, particularly that locally produced, and its constituents, is considered. A few pages are devoted to the measurements since 1957 of radioactive material in the air, both natural particles and pollution attributable to bomb tests; the variation and behaviour of these particles with time and changing meteorological situation is described. The last dozen pages of this volume contain some interesting results regarding the climate of the streets and squares of the city with their strong local differences of temperature, windiness, insolation, reflected radiation and so on. There are some lessons here about the design of streets, and the placing and colour of buildings; and it appears that the virtue of trees in reducing windiness in the streets is considerable.

H.H.L.

The atmospheric chemistry of chlorine and sulphur compounds. Geophysical Monograph No. (3). Ed. by J. P. Lodge. (Washington), American Geophysical Union, 1959. Pp. viii, 129; Figs.; Tables. $5.50.

In the last few years there has been quite a spate of proceedings of symposiums. These volumes are all very much of a pattern. They begin with a photograph, and end with a list, of the delegates : they give *in extenso* the addresses of welcome and the discussions on the papers which were read, but the papers themselves are often reduced to summaries so brief that neither they, nor the discussion which follows, are comprehensible, unless one is prepared to look up original and usually inaccessible references. They are very beautifully printed on heavy glazed paper, handsomely bound and lavishly illustrated : they are expensive and take so long to prepare for publication that the reviews are apt to come out just after the next conference on the same topic, which confuses everybody. And what is it all for ? It makes pleasant reading for the specialist, but something more than this is needed to justify an augmentation of the flood of scientific literature. Such books do not advance scientific knowledge, for they rarely contained anything original : and if their purpose be to help other workers to keep abreast of the latest thought on the subject, it would be better served by a *prompt* and discriminating report in an appropriate Journal.

However, the work under review is the proceedings of a symposium held at the Robert A. Taft Sanitary Engineering Center, Cincinnati, Ohio, on 4-6 November 1957, under the joint sponsorship of the U.S. Public Health Service and the American Geophysical Union. It is an interesting book, largely because, evidently, the symposium was unusually lively and well organized. To begin with, it had a definite purpose – ' to bring together individuals working in the contiguous fields of cloud physics, atmospheric chemistry and

air pollution.' All three groups are interested principally in minor constituents of the atmosphere which originate at the surface, yet so far they have shown little anxiety to collaborate or to learn from each other. Thus, atmospheric chemistry grew up in the pure air of Scandinavia, it is preoccupied with the somewhat ideal concept of the natural atmosphere, and tends to regard industrial pollution as merely the slug in its salad : but sooner or later the atmospheric chemists will have to pay serious attention to pollution. On the other hand, the study of pollution must benefit from what the atmospheric chemists are learning about the natural background of aerosols and trace-gases and about the mechanisms by which such materials are removed from the atmosphere. It is here that the cloud physicist comes into the picture, for precipitation is among the most important of these mechanisms. It is, evidently, very desirable that the three subjects should be closely united, and it was an admirable idea to use this symposium for the purpose.

A second attractive feature of this symposium was that it limited itself to a reasonable number of papers. Twenty-three in three days are, Heaven knows, quite enough : but some recent conferences have galloped through twenty in a day. A consequence of this restraint was that the discussions (which are, after all, the main justification for a symposium) seem to have been fuller and more relaxed, and are certainly easier to follow, than usual.

The first seven papers dealt with analytical methods – a rather specialized field, which one suspects that the cloud physicists found a little cold. The papers bring out sharply the almost complete separation of the three groups of workers with regard to technique. The second section, of four papers, is on ' Field techniques and engineering control.' Two of these, on ' Improved tritilog sensitivity,' conveyed absolutely nothing to the reviewer, partly perhaps because they appear only as very brief summaries. They are followed, however, by an extremely interesting analysis of the output of SO_2 resulting from the burning of fuels in North America, and a discussion of methods of eliminating it. A point that does not readily occur to a meteorologist is that the elimination of SO_2 from smoke would leave us with vast quantities of sulphur compounds on our hands, very difficult to dispose of economically.

The last paper in this section is a study, of a familiar kind, of diffusion of SO_2 from a group of chimneys, and does not appear to have yielded any sensational results.

The third group of papers is headed, rather loosely, ' Atmospheric concentrations.' The first three are on urban pollution and include one excellent paper entitled ' British urban pollution,' by P. J. Lawther. Then follow two papers on the distribution of particulate chloride and sulphate over the U.S.A., and two papers on purely chemical reactions of importance in atmospheric chemistry – one of these being the paper by Junge and Ryan which appeared in *Q.J.*, **84**, No. 1. Oddly sandwiched into this section is a discussion by Fournier d'Albe on the use of salt for the artificial stimulation of rainfall.

Finally, there are four papers dealing with ' The atmospheric budget ' with respect to chlorides and sulphur compounds. This is a short section of twelve pages only, most of it quite familiar.

Pollution is the dominant theme of the whole symposium as one would expect from the venue and sponsorship. Unfortunately the first half of the book is largely preoccupied with the techniques of sampling and analysis, particularly those used in pollution work. This very considerably reduces its interest for the meteorologist. The second half does deal with problems of atmospheric chemistry which are an integral part of meteorology; but of the twelve papers in this section, five are represented only by very brief summaries.

Still, there is much interesting reading here. If we must have reports on symposiums, this is quite a good one. Its general appearance and character have been indicated already. The considerable, and indeed rather excessive reliance on summaries has made it possible to reduce the book to 129 pages, and its list price is $5.50.

B.C.V.O.

Electromagnetic phenomena in cosmical physics. Ed. by B. Lehnert. (Cambridge University Press), 1958. Pp. xiii, 545; Figs.; Tables; Plates. 80s.

The field of magnetohydrodynamics is expanding so rapidly at the present time in a number of different directions that it is of great value to have available this book which gives a complete presentation of the discussions at the International Astronomical Union Symposium No. 6 held at Stockholm in 1956. The book is divided into six parts, Magnetohydrodynamics, Solar electrodynamics, Stellar magnetism, Solar and interplanetary fields, Electromagnetic state in interplanetary space and High-current discharges. Each part contains, on average, 9 papers presented at the symposium. It is impossible to summarize such a comprehensive collection of material and all that is necessary to say is that it is a most invaluable survey of existing theory and experiment in each of the above fields. The editor has wisely included at the end of each paper the subsequent discussion which took place in the symposium. The book, being a product of the Cambridge University Press, is admirably produced.

T.V.D.

Contemporary physics, Vol. 1, No. 1. Taylor and Francis, London, 1959. Pp. 80. 5s.

The first issue of this new journal begins with a short foreword by Sir Cyril Hinshelwood, President of the Royal Society. In it he emphasizes the breadth and depth of modern physics, and the need for experts ' to put the whole field in perspective to a wider circle of their scientific colleagues.'

The title page of the journal tells us that it is a ' journal of interpretation and review ' and the foreword suggests that we can expect general reviews, histories of important developments, discussion of the philosophical foundations of current ideas and the consequences of important discoveries.

The first two articles deal with atomic physics; ' Molecular beams ' by O. R. Frisch and ' The structure of the atomic nucleus ' by R. J. Blin-Stoyle. Although a determined attempt has been made to write for a scientist who is not familiar with current atomic theory, these articles will not be found easy reading by those whose interests lie in other fields.

Geophysics finds a place in this first volume with an interesting account of Alexander von Humboldt and his work, particularly in organizing an international network of observations in terrestrial magnetism. There follow three articles dealing briefly with more limited topics in a manner easily understood by the non-specialist – experiments for teaching electricity, skidding friction and light sources using radioisotopes. The number is completed by reviews, some of considerable length.

Although most scientists look with mixed feelings at any increase in his necessary or potential reading, this is a venture to be welcomed. The price is very reasonable – five shillings per part. There are 80 pages in this first number.

J.S.S.

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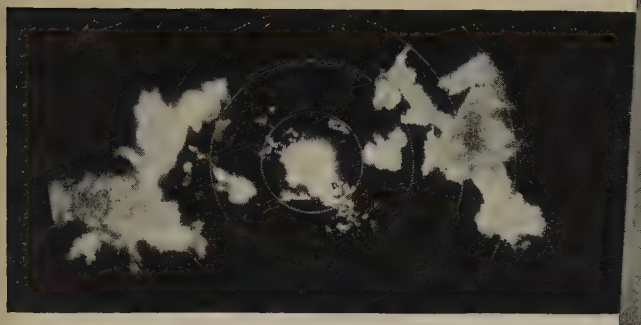

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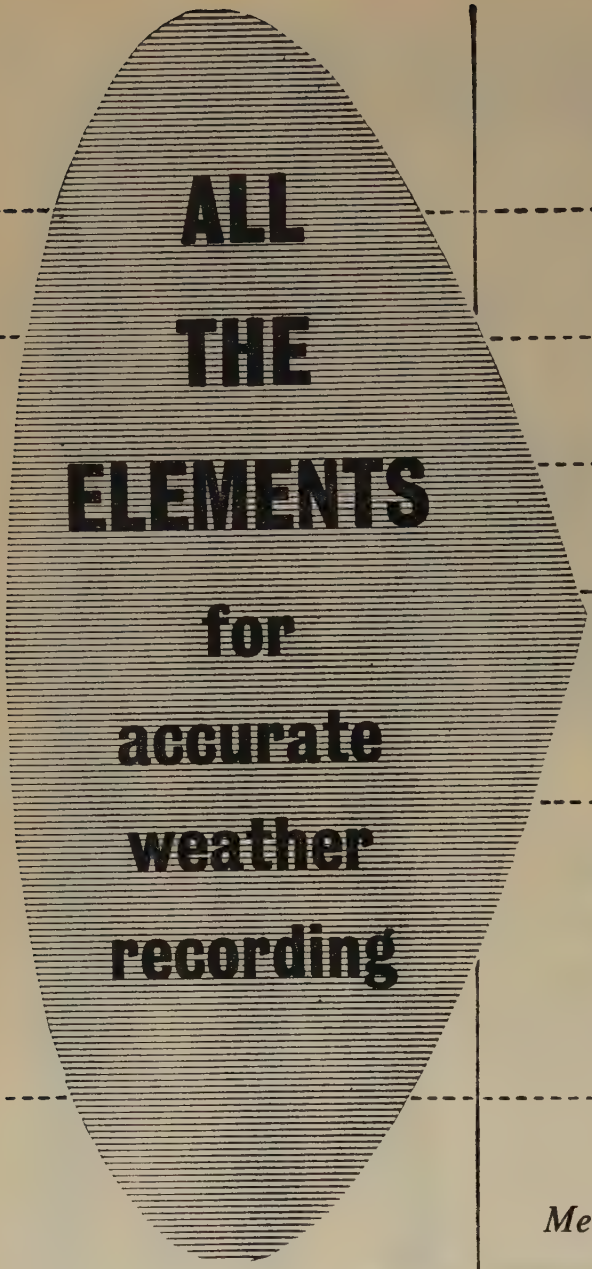

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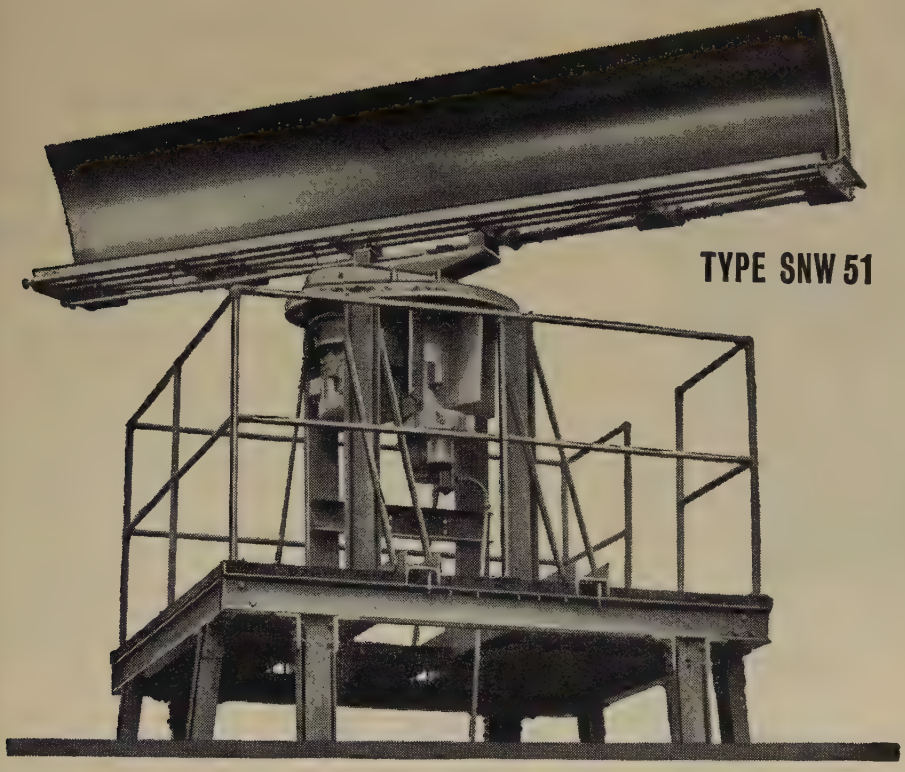

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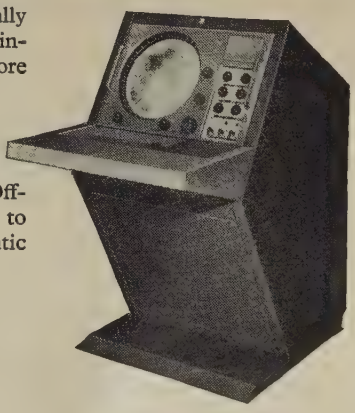

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Published by John Murray

Foreword by R. C. SUTCLIFFE, O.B.E., Ph.D., F.R.S.
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Announcement of third competition

The Royal Meteorological Society announces the third competition for the Napier Shaw Prize. The competition will be open to anyone without restriction of nationality. The prize of £100 is offered on this occasion for an original essay on '**The Upper Atmosphere.**'

For the purpose of this competition the upper atmosphere is regarded as the atmosphere above the tropopause and below the exosphere. Essays on the dynamics, constitution or physics of the atmosphere in this region, or in any part of it, will be accepted, but an essay on purely electrical properties of the ionosphere is not sought. Preference will be given to essays relating two or more aspects of the state of the upper atmosphere. Essays on the inter-relations between the upper atmosphere and the troposphere will also be welcome.

The essay, which must be in English, may include material which the candidate has already published. Essays may be submitted jointly in the names of more than one author.

Candidates are required to submit three copies of their essay, which must be printed or typewritten. In the latter case, only one side of the paper should be used, double spacing should be employed and two of the copies may be good carbon copies. Unless the essay is printed, any mathematics should be written in carefully by hand. Although the scale of the essay will be considerably greater than that of an ordinary paper, candidates are recommended to consult, for general advice, the pamphlet, 'Preparation of papers for the Quarterly Journal.' This may be obtained, free of charge, on application to the Assistant Secretary.

It is required that any essay for which a prize is awarded shall be offered to the Society for publication.

All entries for the competition should be addressed to the Assistant Secretary, Royal Meteorological Society, 49 Cromwell Road, London, S.W.7 and marked clearly 'Napier Shaw Prize.'

Essays for the competition must be received at the Society's Offices *not later* than January 1962.

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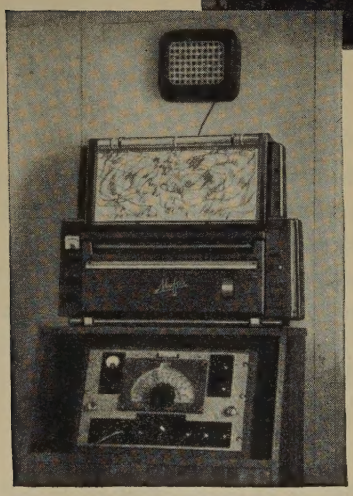

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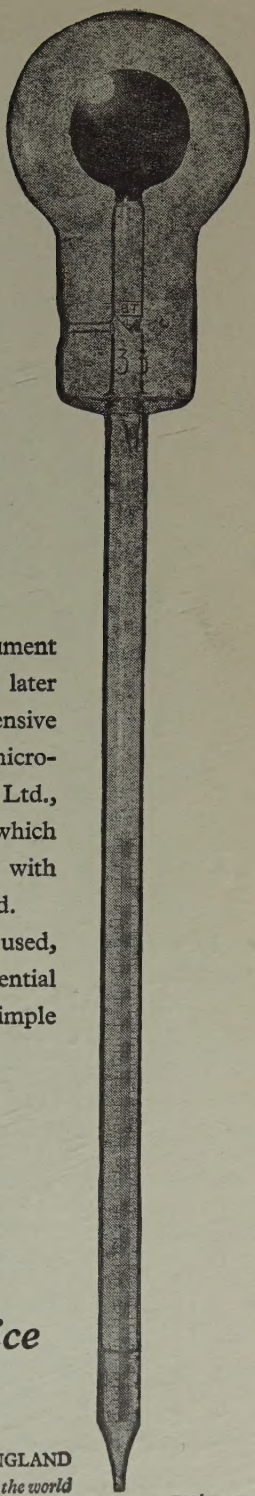

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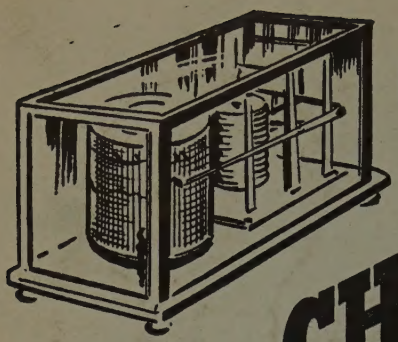

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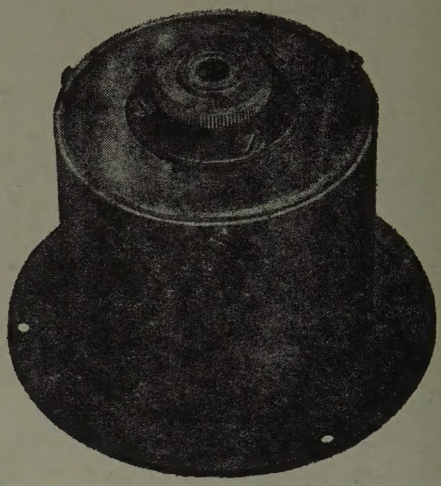

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